MATHEMATICS 1120

Solutions

These solutions do not contain any pictures. You should supply supporting drawings to better understand the solutions.

1. (a) False. The diagonals of a parallelogram which is not a rectangle are not of the same length.
   (b) True. Extend one side of the parallelogram to the outside. The newly created angle is equal to the angle opposite along the shared side, and the two add up to 180 degrees.
   (c) The intended problem, as quoted from the book, is that the midpoints of adjacent sides of the rectangle are to be connected. If that is done, a parallelogram is formed (can you prove it?), but if the rectangle is not a square, the inscribed parallelogram will not be a rectangle, since the angles will not be right angles.

2. (10 points)
   (a) See the textbook, p. 596. The bases should be equilateral triangles.
   (b) There are 4 reflection symmetries and 3 rotational symmetries. There are 3 reflection symmetries about planes containing one side edge and perpendicular to the opposite face, and one reflection symmetry about a plane that cuts the prism in the middle parallel to the bases. The rotation symmetries are 120°, 240°, and 360° about an axis through the centers of the bases.

3. Wall area = 6 ft × 8 ft = (6 · 12) · (8 · 12) = 6912 in². Each tile has area 4 in · 4 in = 16 in². No. of tiles = 6912/16 = 432.

4. Refer to a drawing.
   (a) Let \( h \) denote the length of the unknown length. Then Pythagoras’s Theorem says \( 5^2 + h^2 = 8.5^2 \), or \( h^2 = 8.5^2 - 5^2 = 72.25 - 25 = 47.25 \). So \( h = \sqrt{47.25} = 6.87 \text{ cm} \).
(b) The pentagon consists of 10 triangles congruent to the one formed by line from the center to a vertex, the line to the center of a side, and half a side. Since the line from the center to the center of a side is perpendicular to the side (why?), the area of each of these triangles is \( \frac{1}{2}(5 \cdot 6.87) = 17.175 \). So the area of the pentagon is \( 10 \cdot 17.175 = 171.75 \text{ cm}^2 \).

(c) Volume of \( P = \frac{1}{3}(\text{area of base}) \cdot (\text{height}) = \frac{1}{3}171.175 \cdot 10 = 572.5 \text{ cm}^3 \).

5. (a) 1 gram = 1000 mg, so 1 m = 100 cm, and 1 km = 10,000 cm.
16.8 grams = 16,800 mg 324 cm = 0.00324 km
(c) 1 mile = 8/5 km, so
6 miles = 6 \cdot 8/5 = 48/5 = 9\frac{3}{5} \text{ km}

(d) 0.24 l = 0.24 \cdot 1000 = 240 cm\(^3\) (1 l = 1000 cm\(^3\)).

6. (a) Circumference = \( \pi \cdot 10 \), so area of side = \( \pi \cdot 10 \cdot 12 = 120\pi \). Area of base = \( \pi \cdot 5^2 = 25\pi \). Total area to be painted (inside and out) = \( 2(120 + 25)\pi = 290\pi = 911.06 \text{ ft}^2 \).

(b) 911.06/200 = 4.555 gal.

7. \( F - E + V = 2 \).

(a) \( 20 - E = 12 = 2 \). \( E = 20 + 12 - 2 = 30 \).
(b) \( 12 - 30 + V = 2 \). \( V = 2 - 12 + 30 = 20 \).