1. Let $H$ be a Hilbert space with complete orthonormal set $\{e_n\}_{n=1}^{\infty}$.

(a) Show that $\{e_n\}$ is closed and bounded but not compact.

(b) Let $\delta_n > 0$, $n = 1, 2, \ldots$, and set

$$Q = \{x \in H : x = \sum_{n=1}^{\infty} a_n e_n \text{ with } |a_n| \leq \delta_n, \ n = 1, 2, \ldots \}.$$ 

Then $Q$ is compact if and only if $\sum_{n=1}^{\infty} \delta_n^2 < \infty$. ($Q$ with $\delta_n = 1/n$ is usually called the "Hilbert cube.")
