Director Fields on Flexible Surfaces

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We introduce a nonlinear model for the evolution of biomembranes driven by the $L_2$-gradient flow of a novel elasticity functional describing the interaction of a director field on a membrane with its curvature. Such an interaction gives rise to a spontaneous curvature. In the linearized setting of a graph we present a practical finite element method (FEM), and prove stability and convergence. The FEM deals with the length constraint of director fields while decreasing energy, an idea originally proposed by F. Alouges for finite differences. We extend this approach to the nonlinear model on closed surfaces and introduce a parametric FEM. We present numerical experiments, for both linear and nonlinear models, which agree well with the expected behavior in simple situations and show that defects on the director field may have a dramatic effect on membrane shape. This is joint work with S. Bartels, G. Dolzmann, and A. Raisch.