Asymptotic Analysis in Stochastic Processes, Nonparametric Estimation, and Related Problems:

A Conference in Honor of Rafail Z. Khasminskii on the Occasion of His 75th Birthday

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A New Look at the Stopping Times Related to the Trading Strategies

Vilen Abramov

We provide a probabilistic framework by which an analytic analysis can be performed for the trading strategies under various continuous price dynamics. This, in part, is done by making a connection to the cumulative sum (cusum) procedure of quality control. Although the cusum procedure has been studied quiet extensively, its link with the trading strategies has been mostly untapped. We provide probabilistic characteristics of the trading strategies which one can use to optimize the trading and to price some types of options.

Statistical Mechanics of Ideal Fluid

V.L. Berdichevsky

The central task of turbulence theory is to understand how to average Navier-Stokes equations in case of turbulent motion. It is commonly believed that for the flows away from the walls viscosity is not essential, and the problem is reduced to averaging Euler's equations of ideal fluid. These equations are Hamiltonian, and an assumption of ergodicity of motion allows one to compute average values using Birchhoff-Khinchin's theorem. Such computations have been done for several flows and yield eigenvalue problems similar to that for Schrödinger's equation. In the talk a brief review of this area will be given and the open mathematical problems will be outlined.

References

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Some Rigorous Results on a Stochastic GOY Model

Hakima Bessaih

A stochastic infinite dimensional version of the GOY model is rigorously investigated. Well posedness of strong solutions, existence and p-integrability of invariant measures is proved. Existence of solutions to the zero viscosity equation is also proved. If time permit, some features about the asymptotic exponent of the structure function will be discussed.

On Time Inhomogeneous Controlled Diffusion Processes in Domains

Hongjie Dong

Time inhomogeneous controlled diffusion processes in both cylindrical and non-cylindrical domains are considered. Bellman's principle and its applications to proving the continuity of value functions are investigated.

Effective Macroscopic Dynamics of Stochastic Partial Differential Equations

Jinqiao Duan

An effective macroscopic model for a stochastic microscopic system is derived. The original microscopic system is modeled by a stochastic partial differential equation defined on a domain perforated with small holes or heterogeneities. The homogenized effective model is still a stochastic partial differential equation but defined on a unified domain without holes. The solutions of the microscopic model is shown to converge to those of the effective macroscopic model in probability distribution, as the size of holes diminishes to zero. Moreover, the long time effectivity of the macroscopic system in the sense of convergence in probability distribution, and the effectivity of the macroscopic system in the sense of convergence in energy are also proved.

Practical Semiparametric Bayesian Survival Analysis

Nicholas Oluwaseun Adejuwon Fadipe

In this talk, we investigate the potential of Bayes methods for the analysis of survival data using semiparametric models. Careful applications of some recently popular computational tools, including the sampling based algorithms, are used to find posterior estimates of several quantities of interest even when we are dealing with complex models and unusual data structures. Within the context of survival analysis, our focus is going to be on practical advantages of semiparametric Bayes methods over classical and parametric Bayesian procedures.

Averaging Principle

Mark Freidlin and Alexander Wentzell

Although the Averaging Principle is used in Applied Mathematics about 200 years, even now there are many open questions related to this approach. Around 1960, R.Z. Khasminskii considered averaging for stochastic processes. Some recent results on deterministic and stochastic averaging will be described in the lecture. In particular, we will explain that, even in pure deterministic situation, the averaged motion can be, in a sense, stochastic. Long time behavior of weakly coupled oscillators and close to planar motion of a 3D incompressible fluid will be considered as examples.

Risk Envelopes and Oracle Inequalities

Yuri Golubev

The problem of data-driven choice of smoothing parameter is cornerstone in modern statistics. The main goal in this talk is to provide a brief overview of the method of risk envelopes (MRE) related to this problem. The suggested technique can be viewed a stabilized generalization of Stein's principle of unbiased risk estimation, and we demonstrate numerical advantages of the MRE with simple simulation examples. In order to control statistical performance of the MRE analytically, we obtain the so-called oracle inequalities. As model examples, we consider the spectral cut-off method in inverse estimation, linear functionals estimation, recovering of sparse vectors.

Estimation of Analytic Functions

Ildar Ibragimov

In this talk we consider some problems of statistical estimation of analytic functions. The problems are concerned the unicity theorem for analytic functions. An analytical function is fully determined by its values on an interval. We study how stable is the result with respect to small random perturbations.

Identification Error Bounds and Related Issues

Shaobai Kan

This work consists of two parts. The first part is concerned with identification of systems that are subject to not only measurement noise, but also structural uncertainties such as unmodeled dynamics, nonlinearity mismatch, and bias. Identification errors are analyzed and their dependence on these uncertainties is revealed. In the second part, we study identification problems of systems subject to both deterministic unmodeled dynamics and stochastic observation disturbance, in which the parameter is time-varying (a Markov chain). Error bounds on the persistent identification is given. Online tracking algorithms is also constructed for a slowly varying Markov Chain. [This is a joint work with L.Y. Wang and G. Yin.]

Estimation and Filtering of Smooth Signals for Complete and Partial Observations

Rafail Z. Khasminskii

Overview of results concerning non-parametric estimation of signals with known smoothness observed in the additive White Gaussian Noise (WGN) of small intensity is proposed. The filtering type estimator for this model is also described. Then on-line nonlinear filter for the diffusion observed process and for partially observed signal also are presented. It is proven that the rate of convergence risks to zero for these estimators, as the intensity of WGN tends to zero, is unimprovable in the minimax sense.

Ruin Analysis in the Constant Elasticity of Variance Model

F. Klebaner and R. Liptser

We give results on the probability of absorption at zero of the diffusion process with

non-Lipschitz diffusion coefficient

$$dX_t = \mu X_t dt + \sigma X_t^{\gamma} dB_t,$$

with $X_0 = K$, and $1/2 \leq \gamma < 1$. In finance this is known as the Constant Elasticity of Variance Model and our results give information on the time to ruin $\tau_0 = \inf\{t : X_t = 0\}$. We show that $P(\tau_0 \leq T) > 0$ for all T, give the probability of ultimate ruin, and establish asymptotics

$$\lim_{K\to\infty} \frac{1}{K^{2(1-\gamma)}} \log \mathsf{P}(\tau_0 \leq \mathsf{T}) = -\begin{cases} \dfrac{\mu}{\sigma^2[1-e^{-2\mu(1-\gamma)T}]}, & \mu \neq 0 \\ \dfrac{1}{2\sigma^2(1-\gamma)T}, & \mu = 0. \end{cases}$$

In addition we find an approximation to the most likely paths to ruin. The asymptotics in K is obtained by proving the Large Deviations Principle (LDP) and solving a control problem. The LDP for diffusions with a singular and only Hölder continuous diffusion coefficient requires additional efforts especially due to the absorption at zero of the underlying processes. We give a direct proof of LDP with many details of independent interest.

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Fractional Stability of Diffusion Approximation for Random Differential Equations

Yuriy Kolomiyets

We consider the systems of random differential equations. The coefficients of the equations depend on a small parameter. The first equation, "slow" component, ordinary differential equation, has unbounded highly oscillating in space variable coefficients and random disturbances, which are described by the second equation, "fast" component, with periodic coefficients. The sufficient conditions for weak convergence as small parameter goes to zero of the solutions of the "slow" components to the certain random process are proved.

Maximum Principle for SPDEs and Its Applications

Nicolai V. Krylov

The maximum principle for SPDEs is established in multidimensional C^1 domains. An application is given to proving the Hölder continuity up to the boundary of solutions of one-dimensional SPDEs.

On Krylov's Estimates for Levy Processes and Their Applications to SDEs

Vladimir Kurenok

We prove various versions of Krylov's estimates for solutions of SDEs driven by a Levy process. The obtained estimates generalize those proven originally by N.V. Krylov for the case when the driven process is a Brownian motion. As an application, we prove the existence of solutions for corresponding SDEs driven by Levy processes.

Numerical Approximations for Nonlinear Stochastic Systems with Delays

Harold J. Kushner

We extend the Markov chain approximation numerical methods to controlled general nonlinear delayed reflected diffusion models. The path, control and reflection terms can all be delayed. For the no-delay case, the method is robust and the approximations have physical interpretations as control problems closely related to the original one. These advantages carry over to the delay problem. It is shown how to adapt the methods for getting the approximations, and the convergence proofs are outlined. Extensions to all of the cost functions of current interest as well as to models with Poisson jump terms are possible. A major issue concerns representations of the state that minimize the memory requirements.

Structural Adaptation via L_p-norm Oracle Inequalities

Oleg Lepski

In this paper we study the problem of adaptive estimation of a multivariate function satisfying some structural assumption. We propose novel estimation procedure that adapts simultaneously to unknown structure and the smoothness of the underlying function. The problem of structural adaptation is stated as the problem of selection from a given collection of estimators. We develop a general selection rule and establish for it global oracle inequalities under arbitrary L_p-losses. These results are applied for adaptive estimation in the additive multi–index model.

Liouville Theorem for One Class of the Markov Processes with the Applications to the Functional Differential Equations by Kato, Poincare and Others

Stanislav Molchanov

We are considering the class of the diffusion - jump Markov process on $\mathbb{R}^1$ whose jumps are governed by the random walk on the group of the affine transformations of $\mathbb{R}^1 : x \to ax + b$. The generators of such processes and the corresponding harmonic functions are related to the self- similar differential - functional equations. The different forms of such equations were proposed by Poincare, Birkhoff, Kato and others in the variety of applications(modular functions, vibration of the wires etc.). We proved (together with G. Derfel, Ben Gurion University, Israel) the Liouville theorem (any bounded harmonic function is const.) containing the majority of known results in this area. In some important cases when the set of bounded harmonic functions is not trivial, we provide the description of this set.

An Asymptotic Error Bound for Symmetric Hypothesis Testing in Quantum Statistics

Michael Nussbaum

We consider symmetric hypothesis testing, or discrimination, between two density operators in a finite dimensional quantum setting. For fixed hypotheses, and exponentially decreasing error, we prove a lower bound for the asymptotic risk, in terms of a non-improvable rate exponent. In the case of commuting density operators it coincides with the classical Chernoff bound, where a symmetric analog of the Kullback-Leibler divergence occurs. We give some evidence for the conjecture that our bound is attainable in the general quantum case, and thus represents the optimal rate exponent.

Wave Front Propagation in Randomly Layered Media

George Papanicolaou

In randomly layered media wave pulses propagate according to an elegantly simple law that can be established using a complex martingale for the transmission coefficient. I will introduce this problem and discuss the role of the martingale.

One Class of Multi-dimensional Stochastic Differential Equations Having No Property of the Weak Uniqueness of A Solution

M. Portenko

A standard multi-dimensional Wiener process is at first transformed by an operation of skewing on a given hyperplane, and then the time is randomly changed in such a way that results in a process being the solution to a stochastic differential equation. It turns out that infinitely many transformations of this type lead us to the same equation, and hence this equation has no property of the weak uniqueness of a solution.

On the Replicator Dynamics Behavior under Stratonovich Type Random Perturbations

Nadya Potsepun

We study the behavior of the replicator dynamics systems under White Gaussian Noise (WGN) perturbations in the Stratonovich form. Long-run behavior is described for the system with two pure strategies. Method of Lyapunov functions is used to find the sufficient conditions for asymptotic stability and strong instability of the pure strategies for the system with arbitrary number of pure strategies. Extinction of strictly dominated pure strategies is proven for any level of the noises.

Shift-invariant Second Order Processes

M. M. Rao

It is well-known that (Khintchine or) K-stationary random processes admit shift-invariant operation using which one can obtain a stochastic integral representation for them. Also known is the fact that some processes (such as harmonizable classes) do not admit shift operations. It will be shown that there exist a large class of non-stationary processes that admit shifts. This is a subclass of Karhunen family which also contains the harmonizable classes. Integral representations, through the associated operator families, will be given. This general class is of interest in studying asymptotic distributions of estimators of the associated spectral functions.

A Jump Telegraph Model for Option Pricing to the Black-Scholes Model in Distribution

Nikita Ratanov

A new class of financial market models is proposed. These models are based on generalized telegraph processes: Markov random flows with alternating velocities and jumps occurring when the velocities are switching. While such markets may admit an arbitrage opportunity, the model under consideration is arbitrage-free and complete if directions of jumps in stock prices are in a certain correspondence with their velocity and interest rate behavior. An analog of the Black-Scholes fundamental differential equation is derived, but, in contrast with the Black-Scholes model, this equation is hyperbolic. Explicit formulas for prices of European options are obtained using perfect and quantile hedging. Under some scaling normalization the model converges to the Black-Scholes model in distribution.

Passive Scalar Equation in a Turbulent Incompressible Gaussian Velocity Field

Boris Rozovskii

Time evolution of a passive scalar is considered in a turbulent homogeneous incompressible Gaussian flow. The turbulent nature of the flow results in non-smooth coefficients in the corresponding evolution equation. A strong, in the probabilistic sense, solution of the equation is constructed using Wiener Chaos expansion, and the properties of the solution are studied. Among the results obtained are a certain L_p-regularity of the solution and Feynman-Kac-type, or Lagrangian, representation formula. The results apply to both viscous and conservative flows.

Asymptotically Stationary Processes on Amenable Groups

Bert Schreiber

Vector-valued, asymptotically stationary processes are studied on a σ-compact locally compact amenable group. For such processes, we introduce an associated spectral measure and identify the almost periodic spectrum of their asymptotically stationary covariance and construct a natural, consistent estimator for this almost periodic spectrum.

State Estimation for Probabilistic Discrete Event Systems

Shaolong Shu, Feng Lin, Hao Ying, and Xinguang Chen

A probabilistic discrete event system is a nondeterministic discrete event system where the probabilities of nondeterministic transitions are specified. State estimation problems of probabilistic discrete event systems are more difficult than those of non-probabilistic (or nondeterministic) discrete event systems. In an early paper, we investigated state estimation problems for nondeterministic discrete event systems, where no probabilities are assumed. We defined four types of detectabilities and derived conditions for checking these detectabilities. In this paper, we study state estimation problems for probabilistic discrete event systems. Given a probabilistic discrete event system, we can convert it into a nondeterministic discrete event system by ignoring all probabilities. We first show that if the converted nondeterministic discrete event system is detectable, then the original probabilistic discrete event system is also detectable in a strong sense because it is independent of the probabilities. A more interesting version of detectability is in a weak and probabilistic sense. We define such probabilistic detectabilities in this paper. To find necessary and sufficient conditions for checking probabilistic detectabilities, we need to investigate the convergence of strings, that is, whether we are more and more certain if the system is in a particular state. We derive conditions for a string to converge. We will focus on systems having complete event observation and no state observation. Main results of the paper will be illustrated by examples.

Rate of Convergence of Finite-Difference Scheme for Bellman Equations with Boundary Conditions

Qingshuo Song

This work is concerned with the rate of convergence of finite difference scheme for elliptic Hamilton-Jacobi-Bellman (HJB) equation with boundary condition using probability

method. The corresponding HJB equation is associated to a stochastic control problem with stopping time. The added difficulty is continuity of stopping time with respect to state variable. First, convergence rate on finite difference method for HJB without control is obtained with 1/2, and continuity of stopping time is proved as byproduct. Moreover, the possibility of generalization under control space is investigated. [This is a joint work with G. Yin.]

Near-optimal Hybrid Filtering in a Two-time-scale Model

Qing Zhang

We develop a filtering scheme for hybrid systems with the process dictating the system configuration being a finite-state Markov chain. Exploiting hierarchical structure of the underlying system, the states of the Markov chain are divided into a number of groups so that it jumps rapidly within each group and slowly among different groups. Focusing on reduction of computational complexity, the filtering scheme includes the following steps: (1) Partition the state space of the Markov chain into subspaces, (2) derive a limit system in which the states are averaged out with respect to the invariant distributions of the Markov chain, (3) use the limit system to design quadratic variation test statistics, and (4) use the test statistics to identify which ergodic class the aggregated process belongs to and to construct near-optimal filter. For demonstration, a numerical example is also presented. [This is a joint work with Jianwu Wang and G. Yin.]

Stability of Regime-Switching Diffusions

Chao Zhu

This work is devoted to stability of regime-switching diffusion processes. After presenting the formulation of regime-switching diffusions, the notion of stability is recalled, and necessary conditions for p-stability are obtained. Then results on stability and instability for systems arising in approximation are presented. Easily verifiable conditions are established. As a by-product, necessary and sufficient conditions for existence of solutions of certain systems of algebraic equations are also given using a finite-state Markov chain. An example is examined as a demonstration. A remark on linear (in the continuous component) systems is also provided. [This is a joint work with Rafail Z. Khasminskii and G. Yin.]