Homework 3
Due January 30, 2007

Turn in:

1. Let V and W be vector spaces over a field F. A *homomorphism* from V to W is an F-linear transformation, i.e., a function $\phi : V \to W$ such that $\phi(v_1 + v_2) = \phi(v_1) + \phi(v_2)$ and $\phi(\alpha v) = \alpha\phi(v)$.

An *isomorphism* is a homomorphism that is one-to-one and onto. Prove that the inverse of an isomorphism is an isomorphism.

2. Prove that $\mathbb{R}$ is a vector space over $\mathbb{Q}$, where addition is the usual addition and scalar multiplication is the usual multiplication.

For which values of x is the set $\{1, x\}$ linearly independent in this vector space?

Do not turn in:

3. Suppose that v_1, v_2, and v_3 are linearly independent. Prove that $v_1 + v_2$, $v_2 + v_3$, and $v_3 + v_1$ are also linearly independent.

4. Let V be a vector space over F. Prove that $0 \cdot v = 0$ for all v in V. Prove that $-(\alpha v) = (-\alpha v)$ for all v in V and all α in F.