Write your solutions in a blue book. To receive full credit you must show all work. You are allowed to use an approved graphing calculator unless otherwise indicated. There are 15 problems worth a total of 200 points. The time limit is 2\frac{1}{2} hours.

1. (10 points) Use the definition of the derivative to differentiate the following function.
   \[ f(x) = \frac{x}{1 - 3x} \]

2. (7 points each) Find the exact value of each of the following limits. Write “\(\infty\),” “\(-\infty\),” or “does not exist” if appropriate. It is particularly important to show your work on this problem.
   (a) \(\lim_{x \to 1} \frac{3x - 3}{x^2 + 5x - 6}\)
   (b) \(\lim_{x \to -\infty} \left( e^x + \frac{x - 7}{13x + 5} \right)\)
   (c) \(\lim_{x \to -\infty} \left( \sqrt{3x^2 + x} - \sqrt{3} \cdot x \right)\)

3. (8 points each) Differentiate the following functions.
   (a) \(f(x) = x^3 \ln x\)
   (b) \(g(x) = \frac{e^x}{\arctan x}\)
   (c) \(h(x) = \cos\left(\tan(\sqrt{x})\right)\)

4. Evaluate the following integrals.
   (a) (8 points) \(\int \frac{x^3 + 2x^2 + 3x - 4}{x^2} \, dx\)
   (b) (8 points) \(\int x^2 e^{x^3} \, dx\)
   (c) (9 points) \(\int_0^8 \left( x + \cos(2\pi x) \right) \, dx\)

5. (10 points) Find \(\frac{dy}{dx}\) and \(\frac{d^2 y}{dx^2}\) at the point (2,1) on the curve \(x^2 + 2xy - y^2 = 7\).
6. (10 points) A block of ice maintains the shape of a cube as it melts at the constant rate of 3 cubic inches per minute. Find the rate at which the surface area of the cube is changing when the side of the cube is 12 inches long. Hint: Recall that a cube has six faces.

7. (10 points) A ball is thrown directly downward with initial downward velocity \( V \) from a point 144 feet above the ground. It accelerates downward at the rate of 32 feet per second per second.

(a) Find the height \( h \) of the ball \( t \) seconds after it is released. Express \( h \) in terms of \( V \).

(b) Find the initial velocity \( V \) if the ball reaches the ground in exactly 2 seconds.

8. (10 points) A certain government decides at 8 AM on January 1 to liquidate (sell) its holdings of US treasury bills continually at the rate of \( R \) dollars worth of bills per hour where \( R = 10,000,000 + 600,000t^2 \), where \( t \) is the time measured in hours, starting with \( t = 0 \) at 8 AM. Find the total value of the bills it liquidates by 10 AM on January 3, fifty hours later.

9. (10 points) The volume \( V \) of a spherical balloon is given by the formula \( V = \frac{4\pi r^3}{3} \) where \( r \) is the radius of the balloon. When \( r \) is given in inches, \( V \) will be in cubic inches.

(a) Find the average rate of change in the volume of the balloon, as a function of its radius, as the radius is increased from \( r = 10 \) inches to \( r = 12 \) inches. Be sure to give proper units.

(b) Find the instantaneous rate of change in the volume of the balloon as a function of its radius when \( r = 11 \) inches. Be sure to give proper units.

10. (10 points) A continuous, increasing function \( f \) takes on the values indicated in the table below. Use a Riemann sum to give a lower estimate for \( \int_{0}^{15} f(x) \, dx \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>10</td>
<td>15</td>
<td>18</td>
<td>20</td>
<td>30</td>
<td>40</td>
</tr>
</tbody>
</table>
11. (10 points) The graph of a function $f$ is shown below.

For each condition listed, find all values of $x$ in the interval $[-4, 4]$ that have the stated property.

(a) $f'(x) > 0$.
(b) $f'(x) = 0$.
(c) $f'(x)$ does not exist.

12. (10 points) The function $g$ is defined by $g(x) = \int_{0}^{x} f(t) \, dt$, $0 \leq x \leq 10$ where the graph of $f$ is shown.

Sketch the graph of $g$. Be sure to indicate concavity, critical points and intervals of increase and decrease.
13. (10 points) Find the maximum and minimum values of the function
\[ h(x) = 2x^3 - 33x^2 + 144x \]
on [0, 10].

14. (10 points) Sketch the graph of ONE function \( f \) that has ALL of the following properties.

(i) \( f(x) \) is defined for all real numbers \( x \).
(ii) \( f \) is not continuous at \( x = 5 \), but \( f \) is continuous at all other points.
(iii) \( f \) is not differentiable at \( x = 1 \) or \( x = 5 \), but \( f \) is differentiable at all other points.
(iv) \( f \) is increasing on \([1, 5)\)
(v) \( \lim_{x \to -\infty} f(x) = -3 \)
(vi) \( \lim_{x \to 1} f(x) = 2 \)
(vii) \( \lim_{x \to 5^-} f(x) = \infty \)
(viii) \( \lim_{x \to 5^+} f(x) = -\infty \)
(ix) \( \lim_{x \to \infty} f(x) = 4 \)

15. (20 points) Sketch the graph of the function \( f \) on the interval \([0, 2\pi]\),
based on the following information. Show intervals of increase and decrease, and concavity. Label all local maxima and minima and points of inflection.

(a) \( f'(x) = \sin x + \frac{1}{2} \)
(b) \( f''(x) = \cos x \)
(c) \( f(0) = 1 \)
(d) \( f(\pi) \approx 4.57 \)
(e) \( f(2\pi) \approx 4.14 \)