

FINAL EXAMINATION, MAT 2010
April 26, 2007

Write your solutions in a blue book. To receive full credit you must show *all* work. You are allowed to use an *approved* graphing calculator unless otherwise indicated. There are 15 problems worth a total of 200 points. The time limit is $2\frac{1}{2}$ hours.

1. (10 points) Use the *definition* of the derivative to differentiate the following function.

$$f(x) = \sqrt{x+1}$$

2. (7 points each) Find the exact value of each of the following limits. Write " ∞ ," " $-\infty$," or "does not exist" if appropriate. It is particularly important to show your work on this problem.

(a) $\lim_{x \rightarrow \infty} \frac{x^2}{e^x - 50}$

(b) $\lim_{x \rightarrow \infty} \sqrt{2x^2 + 3x + 4} - \sqrt{2} x$

(c) $\lim_{x \rightarrow 1^+} \frac{1}{x^3 - 3x + 2}$

3. (8 points each) Differentiate the following functions.

(a) $f(x) = e^{4x^2}$

(b) $g(x) = x^3 \ln x$

(c) $h(x) = \frac{\sin x}{\cos x}$

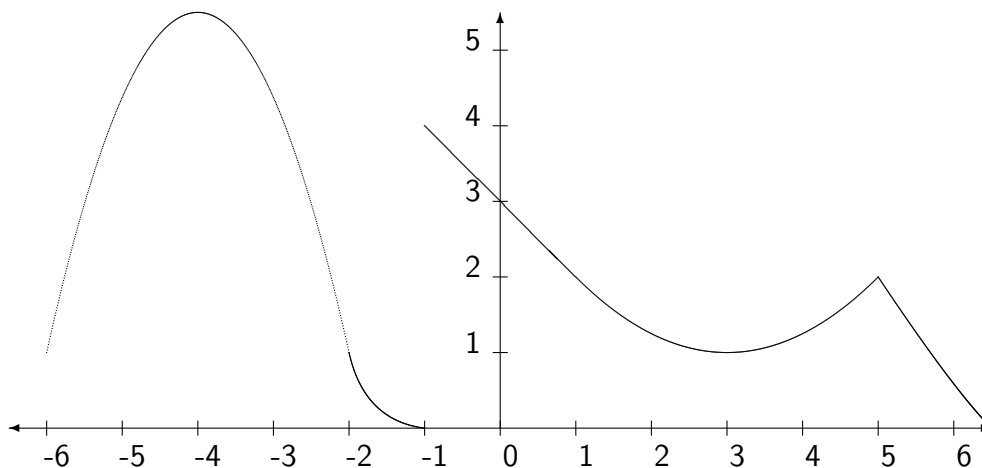
4. Evaluate the following integrals.

(a) (8 points) $\int e^{3x} - \cos x + \sin 2x \, dx$

(b) (8 points) $\int \frac{3}{x^2 + 1} - \frac{1}{x} \, dx$

(c) (9 points) $\int_1^3 (x-1)^2 \, dx$

5. (10 points) The graph of a function f is shown below.



- (a) For what values of x in the interval $(-6, 6)$ is $f'(x) = 0$?
 - (b) For what values of x in the interval $(-6, 6)$ is $f'(x) > 0$?
 - (c) For what values of x in the interval $(-6, 6)$ is $f'(x) < 0$?
 - (d) Where in the interval $(-6, 6)$ is $f'(x)$ at a minimum?
6. (8 points) If $z^2 = x^2 + y^3$, $dy/dt = -2$, and $dz/dt = 3$, find dx/dt when $x = 10$, $y = 5$, and $z = 15$.
7. (12 points) Find the points (x, y) on the ellipse $9x^2 + 4y^2 = 9$ for which the distance from (x, y) to $(-1, 0)$ is as large as possible.
8. (10 points) Use a differential to estimate $\sqrt{25.3}$.
9. (10 points) An automobile driver applies the brakes to slow down. The speed of the automobile is $\frac{140}{t+1}$ feet per second at time t , the number of seconds into the deceleration. Use a Riemann sum with at least 4 terms to give an upper estimate for the distance the automobile travels during the first 12 seconds the driver is slowing down. Be sure to use proper units for your answer. (Your estimate should be larger than the actual distance traveled.)

10. (10 points) Let $f(x)$ be defined as follows

$$f(x) = \begin{cases} 6 - x^2 & \text{if } x \leq -2 \\ |x| & \text{if } -2 < x \leq 2 \\ x + 1 & \text{if } x > 2 \end{cases}$$

- (a) Find all points in the interval $(-5, 5)$ where f is not continuous.
- (b) Find all points in the interval $(-5, 5)$ where f is not differentiable.
11. (10 points) The temperature of Lake Erie on April 26 this year is 38.0 degrees Fahrenheit. This temperature is projected to increase. Measuring time t in years, at time t the instantaneous rate of increase is projected to be $0.0023t$ degrees per year over the next century. Find the projected temperature of Lake Erie on April 26 one hundred years from now.
12. (10 points) An unauthorized technician is adjusting the height of a loudspeaker to affect the number of people in the crowd who can actually hear it. The percentage of people who can hear the loudspeaker is

$$\frac{5000}{(x - 10)^2 + 100}$$

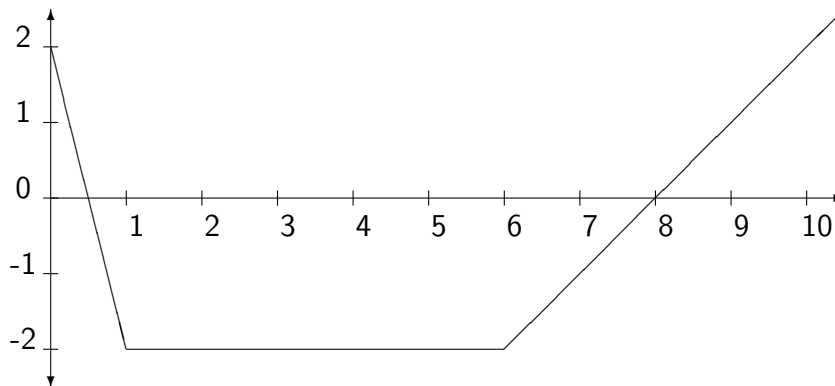
where x is the height, in feet, of the loudspeaker above the ground.

Be sure to state your answers below in proper units.

- (a) Find the average rate of change in the percentage of people who can hear the loudspeaker when it is raised from 10 feet above the ground to 20 feet above the crowd.
- (b) Find the instantaneous rate of change in the number of people among a crowd of 10,000 who can hear if the loudspeaker is raised from a height of 15 feet.
13. (10 points) The [differentiable] function f satisfies the following conditions.
- (i) $f''(x) = e^x$ for all real numbers x .
- (ii) $f'(0) = -1$
- (iii) $f(0) = 4$

Find $f(1)$.

14. (10 points) The function F is defined by $F(x) = \int_0^x f(t) dt$, $0 \leq x \leq 10$ where the graph of f is shown.



- (a) Find $F(0)$.
- (b) Sketch the graph of F . Be sure to indicate critical points and intervals of increase and decrease.
- (c) Find $F'(2)$.
15. (20 points) The function f satisfies the following conditions.
- $f(x)$ is defined for all real numbers x .
 - f is periodic with period 2π .
 - $f'(x) = \cos x - \sin x$
 - $f(0) = 3$

Compute $f''(x)$, then sketch the graph of the function f on the interval $[0, 2\pi]$, clearly indicating intervals of increase and decrease, critical points, points of inflection, and correct concavity.