Complex Numbers

Complex numbers are expressions of the form $a + bi$, where a and b are ordinary real numbers. For example, $3+2i$ and $\sqrt{2}-\frac{1}{2}i$ are complex numbers.

In this number system, you can add, subtract, multiply, and divide. Let's review how these operations work.

Addition (or subtraction) is easy: just add (or subtract) the "real" and "imaginary" parts separately. For example,

$$(3 + 2i) + (4 - 5i) = (3 + 4) + (2i - 5i) = 7 - 3i.$$

Problem 1. $(2 + 3i) + (4 + 5i)$

Problem 2. $(3 + 3i) - (6 - 4i)$

Problem 3. $(-2 + -\frac{1}{2}i) + (\frac{1}{3} + 2i)$

Multiplication of complex numbers is a bit trickier. You have to use FOIL (that is, the distributive law), and you also have to remember that $i^2 = -1$. For example,

$$\begin{aligned}(3 + 2i)(4 + 5i) &= 3 \cdot 4 + 3 \cdot 5i + 2i \cdot 4 + 2i \cdot 5i = 12 + 15i + 8i + 10i^2 \\ &= (12 - 10) + (15i + 8i) = 2 + 23i.\end{aligned}$$

Problem 4. $(3 + 3i)(6 - 4i)$

Problem 5. $(8 - 2i)(1 + i)$

Problem 6. $(5 + 3i)(5 - 3i)$

Problem 6 shows an important property of complex numbers. If you multiply $a + bi$ with its "conjugate" $a - bi$, then you will always obtain an ordinary real number as the answer. This is important for division of complex numbers. In order to divide one complex number by another, multiply the numerator and denominator by the conjugate of the denominator. For example,

$$\frac{1 + i}{4 + 3i} = \frac{(1 + i)(4 - 3i)}{(4 + 3i)(4 - 3i)} = \frac{4 - 3i + 4i - 3i^2}{16 - 12i + 12i - 9i^2} = \frac{7 + i}{25} = \frac{7}{25} + \frac{1}{25}i.$$

Problem 7. $\dfrac{8-2i}{6-4i}$

Problem 8. $\dfrac{4+5i}{2+3i}$

Problem 9. $\dfrac{-4-2i}{1+2i}$