Graphs That Are Randomly Traceable from a Vertex

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Abstract

A graph G is *randomly traceable* from one of its vertices v if every path in G starting at v can be extended to a hamiltonian path of G that starts at v. A complete classification of these graphs will be given, and some corollaries will be stated. Also, a few related concepts will be briefly discussed.

Introduction

This paper will develop the notion of graphs that are randomly traceable from a vertex. A key theorem will provide the mechanism for completely characterizing these graphs. Then, several results about these graphs can be derived. Finally, a few related topics will be mentioned briefly.

Throughout this paper, it will be assumed that all graphs mentioned are connected.

Chartrand and Kronk [ref1] defined a graph G to be *randomly traceable* if every path in G can be extended to a hamiltonian path of G. This idea can be modified by localizing it to a particular vertex.

Definition 1. If G is a graph and v is a vertex of G, then G is *randomly traceable* from v if every path in G starting at v can be extended to a hamiltonian path in G starting at v.

Clearly, a graph is randomly traceable if and only if it is randomly traceable from every vertex.

Define a maximal path from v to be a path that cannot be extended. Then Definition 1 is equivalent to the fact that every maximal path from v is a hamiltonian path.

Intuitively, this means that one can start at v, travel to any adjacent vertex that has not already been visited, and repeat this process until it is impossible to continue. At this point, a hamiltonian path will have been constructed. In other words, no matter how hard one tries, it is impossible to fail to construct a hamiltonian path as long as one starts at v. This motivation explains the relevance of the terminology.

In Figure 1, G is randomly traceable from v_1 and v_2. Also, H is randomly traceable from v, u_1, and u_2.

Figure 1. Some graphs that are randomly traceable from a vertex.

A colleague has called this question the "drunken salesman" problem. Even if the salesman does not know where he is going, he will always visit each location exactly once as long as he is traveling on a graph that is randomly traceable from v, where v is his initial location. However, the salesman must be conscious enough not to return to a place that he has already visited.

Perhaps a slightly more accurate description would be the "nearsighted salesman" problem. At any given time, the salesman can only see the locations that are adjacent to him. However, he does not need any more information. He will never be forced to return to a location that he has already visited if he is traveling on a graph that is randomly traceable from v, where v is his initial location.

Key Theorem

Lemma 2. *G is randomly traceable from v if and only if for all vertices u of G such that u is adjacent to v, $G - v$ is randomly traceable from u.*

Proof. Suppose that G is randomly traceable from v. Let u be any vertex adjacent to v, and let P be a path in $G - v$ starting at u. Then vP is a path in G. Therefore vP is extendable to a hamiltonian path Q since G is randomly traceable from v. So, $Q - v$ is a hamiltonian path of $G - v$ that extends P. Hence $G - v$ is randomly traceable from u.

Now suppose that for all $u \in V(G)$ such that u is adjacent to v, $G - v$ is randomly traceable from u. If G has only one vertex, then $G \cong K_1$, so G is randomly traceable.

Suppose that G has more than one vertex. Let P be any path in G starting at v. If P has non-zero length, then $P - v$ is a path in $G - v$ that starts at a neighbor of v. Hence $P - v$ is extendable to a hamiltonian path Q of $G - v$. Therefore vQ is a hamiltonian path of G that extends P.

If P has length 0, then P can be extended to any neighbor of v. Then this path can be extended to a hamiltonian path by the argument of the previous paragraph. □

Definition 3. G is a *parent* of H and H is a *child* of G if there exists a vertex v of H such that $H - v$ is isomorphic to G and H is randomly traceable from v.

In Figure 1, H is a child of G.

Definition 4. G is an *ancestor* of H and H is a *descendant* of G if there exists a sequence of graphs $G_1, G_2, \ldots, G_n$, where $n \geq 1$, such that $G = G_1$, $H = G_n$, and for all i such that $1 \leq i \leq n-1$, G_i is a parent of G_{i+1}.

Note that if G is any graph, then G is a descendant of itself.

Theorem 5. *G is randomly traceable from a vertex if and only if G is a descendant of K_1.*

Proof. Suppose that $G \cong K_1$. By inspection, G is randomly traceable from its only vertex.

Suppose that G is a descendant of K_1 and that $G \ncong K_1$. Then G is the child of some graph and so is randomly traceable by definition.

Now suppose that G is randomly traceable from a vertex. If G has only one vertex, then $G \cong K_1$.

Let G have n vertices, where $n \geq 2$, and let $v \in V(G)$ such that G is randomly traceable from v. Then G is a child of $G - v$. By Lemma 2, $G - v$ is randomly traceable from the vertices that are adjacent to v in G. Also, $G - v$ has $n - 1$ vertices.

By induction on n, G must be a descendant of K_1. □

This theorem will be the basis for the classification of graphs that are randomly traceable from a vertex. Using it, the search is reduced to finding all descendants of K_1.

Infertile Descendants

Definition 6. If G is randomly traceable from more than one vertex or $G = K_1$, then G is *fertile*. If G is randomly traceable from a unique vertex and $G \neq K_1$, then G is *infertile*. If v is the unique vertex from which G is randomly traceable, then G is *infertile* at v.

Note that *infertile* is not equivalent to *not fertile* because not all graphs are randomly traceable from a vertex.

Lemma 7. *If $v \in V(G)$ and $\deg v = 1$, then either G is a path or G can only be randomly hamiltonian from v.*

Proof. Let G be a graph with vertex v such that $\deg v = 1$. Suppose that G is randomly traceable from some other vertex w. Let P be a shortest path from w to v. Then P is maximal since $\deg v = 1$. Hence P must be a hamiltonian path, so it passes through every vertex. Therefore G is a path. □

The motivation for these definitions can be seen in the following theorem.

Theorem 8. *If G is infertile, then G has a unique infertile child.*

Proof. Let G be infertile, and let v be the unique vertex from which G is randomly traceable. Then G is not a path since paths are randomly traceable from both of their endpoints. Let H be a child of G. Then H can be formed by adding a vertex u and attaching it to some of the vertices of G. Because G is infertile at v, the only possible neighbor of u is v. Thus G has a unique child H, which is constructed by adding a vertex u and the edge uv. H is not a path since G is not a path, and $\deg u = 1$. Therefore H is infertile by Lemma 7. □

Note that this theorem implies that every descendant of an infertile graph is infertile and has a unique child. This shows that it is easy to find the descendants of infertile graphs. Also, all infertile graphs can be generated from their fertile ancestors. Therefore an investigation of fertile graphs will lead to a complete understanding of the family tree.

Fertile Children of Particular Graphs

This section will exhibit the children of particular graphs that are randomly traceable from a vertex.

Lemma 9. *If $n \geq 2$ and G is a child of P_n, then G is isomorphic to P_{n+1} or C_{n+1}.*

Proof. P_n is randomly traceable only from its endpoints, so the only children of P_n are P_{n+1} and C_{n+1}. □

Lemma 10. *If $n \geq 5$ and G is a fertile child of C_n, then G is isomorphic to $C_n + vx + vz$ or $C_n + vx + vy + vz$, where $v \notin V(C_n)$ and xyz is an arc of C_n. Also, both of these graphs have no fertile children.*

Proof. C_n is randomly traceable from all of its vertices, so the children of C_n are formed by adding a new vertex v and attaching it to some of the vertices of C_n. Let G be such a child. Let the vertices of C_n be labelled $v_1, v_2, \ldots, v_n$ such that v_n is adjacent to v_1 and for all i such that $1 \leq i \leq n-1$, v_i is adjacent to v_{i+1}.

There must be at least one neighbor of v, so assume that v is adjacent to v_1. If v_1 is the only neighbor of v, then G is infertile because $\deg v = 1$.

Now suppose that v has more than one neighbor. Suppose that v is also adjacent to v_j, where $4 \leq j \leq n-1$. Let v_i be any other vertex where $i \leq j$. Without loss of generality, it can be assumed that $i \leq \frac{j+1}{2}$. Then $v_i v_{i-1} \ldots v_1 v v_j v_{j-1}$ is not extendable to a hamiltonian path because it is not extendable to v_n. Therefore G can only be randomly traceable from v and v_k, where $j < k \leq n$.

Similarly, if $3 \leq j \leq n-2$, then G can only be randomly traceable from v and v_k, where $2 \leq k < j$. Hence G is infertile if $4 \leq j \leq n-2$.

Thus if v has two neighbors u and w such that the distance in C_n from u to w is at least 3, then G is infertile. Also, if the distance from u to w is exactly 2, then G can only be randomly traceable from v and y, where y is the common neighbor in C_n of u and w.

Hence there are only three remaining possibilities. The first possibility is that v is adjacent only to two consecutive vertices in C_n. Without loss of generality, assume that these vertices are v_1 and v_n. Let v_i be any vertex other than v. It can be assumed that $i \leq \frac{n+1}{2}$. Then the path $v_i v_{i-1} \ldots v_1 v_n v$ cannot be extended to a hamiltonian path because it is not extendable to v_{n-1}. Thus G is infertile.

The second and third possibilities are that v is adjacent only to two vertices x and z which have a common neighbor y in C_n, or that v is adjacent to three consecutive vertices of C_n. In these cases, $G \cong C_n + vx + vz$ or $G \cong C_n + vx + vy + vz$, where xyz is an arc of C_n. Either way, G is randomly traceable exactly from v and y because the distance from x to z in C_n is exactly 2. Each of these graphs has exactly two children, all of which can be seen to be infertile by inspection. □

Lemma 11. *If $n \geq 4$ and G is a fertile child of K_n, then G is isomorphic to K_{n+1} or $K_{n+1} - uv$, where u and v are any two vertices of K_{n+1}. Also, $K_{n+1} - uv$ has no fertile children.*

Proof. K_n is randomly traceable from all of its vertices. Therefore the children of K_n are formed by adding a new vertex v and attaching it to some of the vertices of K_n. Let G be such a child, and let the vertices of K_n be v_1, v_2, ..., v_n.

Suppose that v is not adjacent to a vertex of K_n. Without loss of generality, let this vertex be v_n. Let w be any other vertex of K_n. Without loss of generality, let $w = v_1$. Then the path $v_1 v_2 \ldots v_n$ is not extendable to a hamiltonian path because it cannot be extended to v. So G is not randomly traceable from w.

Therefore if v is not adjacent to a vertex u of K_n, then G can only be randomly traceable from v and u. Hence if v is not adjacent to two or more vertices of K_n, then G is infertile.

If v is adjacent to every vertex of K_n, then $G \cong K_{n+1}$. If v is adjacent to every vertex but u, then $G \cong K_{n+1} - uv$, and G is randomly traceable only from v and u.

$K_{n+1} - uv$ has exactly two children, both of which can be seen to be infertile by inspection. □

Lemma 12. *If $n \geq 2$ and G is a fertile child of $K_{n,n}$, then G is isomorphic to $K_{n,n+1}$ or $K_{n,n+1} + uv$, where u and v are vertices of the larger partite set.*

Proof. $K_{n,n}$ is randomly traceable from all of its vertices, so the children of $K_{n,n}$ are formed by adding a new vertex v and attaching it to some of the vertices of $K_{n,n}$. Let G be such a child, and let the partite sets of $K_{n,n}$ be X and Y, where $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$.

Suppose that v is not adjacent to some vertex in Y. Without loss of generality let this vertex be y_n. Let x be any vertex of X. Without loss of generality, let $x = x_1$. Then the path $x_1 y_1 x_2 y_2 \ldots x_n y_n$ is not extendable to a hamiltonian path since it is not extendable to v. So G is not randomly traceable from x.

Hence if v is not adjacent to all of the vertices in Y, then G is not randomly traceable from any of the vertices in X. Similarly, if v is not adjacent to all of the vertices in X, then G is not randomly traceable from any of the vertices of Y. So if v is not adjacent to at least one vertex in Y and one vertex in X, then G is infertile.

Suppose that v is adjacent to at least two vertices in Y. Without loss of generality, let these vertices be y_1 and y_2. Then the path $y_1 v y_2 x_2 y_3 x_3 \ldots y_n x_n$ is not extendable to a hamiltonian path because it cannot be extended to x_1. So G is not randomly traceable from y_1. Similarly, G is not randomly traceable from y_2. Now let y be any other vertex in Y. Without loss of generality, let $y = y_n$. Then the path $y_n x_n y_{n-1} x_{n-1} \ldots y_3 x_3 y_2 v y_1 x_1$ is not extendable to a hamiltonian path because it cannot be extended to x_2. So G is not randomly traceable from y.

Thus G is not randomly traceable from any of the vertices in Y if v is adjacent to at least two vertices of Y. Similarly, if v is adjacent to at least two vertices of X, then G is not randomly traceable from any of the vertices of X.

Therefore if v is adjacent to at least two vertices in Y and two vertices in X, then G is infertile.

There are only two remaining possibilities for G to be fertile. The first possibility is that v is adjacent to all of the vertices in Y and none of the vertices in X. Then $G \cong K_{n,n+1}$, and G is randomly traceable only from v and the vertices of X.

The other possibility is that v is adjacent to all of the vertices in Y and exactly one vertex in X. Let this vertex be u. Then $G \cong K_{n,n+1} + uv$, where u and v are vertices of the larger partite set. In this case, G is randomly traceable only from v and the vertices of X. □

Lemma 13. *If $n \geq 2$ and G is a fertile child of $K_{n,n} + uv$, where u and v are vertices of the same partite set, then G is isomorphic to $K_{n,n+1} + uv$, where u and v are vertices of the larger partite set.*

Proof. Let X and Y be the two partite sets of $K_{n,n}$, where u and v are vertices in X. Then $K_{n,n} + uv$ is randomly traceable only from the vertices of Y, so the children of $K_{n,n} + uv$ are formed by adding a new vertex w and attaching it to some of the vertices of Y. Let G be such a child.

Suppose that w is not adjacent to every vertex in Y. Then, w is not adjacent to at least one vertex in X and one vertex in Y. As in the proof of Lemma 12, G is infertile.

Hence if G is fertile, then w is adjacent to every vertex in Y. In this case, $G \cong K_{n,n+1} + uv$, where u and v are vertices of the larger partite set. □

Lemma 14. *If $n \geq 2$ and G is a fertile child of $K_{n,n+1}$, then G is isomorphic to $K_{n+1,n+1}$ or $K_{n+1,n+1} - uv$, where u is a vertex of one partite set and v is a vertex of the other partite set. Also, $K_{n+1,n+1} - uv$ has no fertile children.*

Proof. Let X and Y be the two partite sets of $K_{n,n+1}$, where X is the larger set. Let $X = \{x_1, x_2, \ldots, x_{n+1}\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$. Then $K_{n,n+1}$ is randomly traceable only from the vertices in X. Therefore the children of $K_{n,n+1}$ are formed by adding a new vertex v and attaching it to some of the vertices in X. Let G be such a child.

Suppose that v is not adjacent to a vertex in X. Without loss of generality, let this vertex be x_{n+1}. Let x be any other vertex in X and y be any vertex in Y. Without loss of generality, let $x = x_1$ and $y = y_1$. Then the paths $x_1y_1x_2y_2\ldots x_ny_nx_{n+1}$ and $y_1x_1y_2x_2\ldots y_{n-1}x_{n-1}y_nx_{n+1}$ cannot be extended to hamiltonian paths since they cannot be extended to v. So G is not randomly traceable from x or y.

Therefore if v is not adjacent to a vertex u in X, then G can only be randomly traceable from v and u. Hence if v is not adjacent to at least two vertices in X, then G is infertile.

There are only two remaining possibilities for G to be infertile. The first possibility is that v is adjacent to every vertex in X. Then $G \cong K_{n+1,n+1}$.

The second possibility is that v is adjacent to all but one vertex in X. Let this vertex be u. Then, $G \cong K_{n+1,n+1} - uv$, where u is a vertex of one partite set and v is a vertex of the other partite set. In this case, G is randomly traceable only from u and v.

$K_{n+1,n+1} - uv$ has exactly two children, both of which can be seen to be infertile by inspection. □

Lemma 15. *If $n \geq 2$ and G is a fertile child of $K_{n,n+1} + uv$, where u and v are vertices of the larger partite set, then G is isomorphic to $K_{n+1,n+1} + uv$, where u and v are vertices of the same partite set.*

Proof. Let X and Y be the partite sets of $K_{n,n+1}$, where X is the larger partite set. Let $X = \{u, v, x_1, x_2, \ldots, x_{n-1}\}$ and let $Y = \{y_1, y_2, \ldots, y_n\}$. Then $K_{n,n+1} + uv$ is randomly traceable only from the vertices of X, so the children of $K_{n,n+1} + uv$ are formed by adding a new vertex w and attaching it to some of the vertices of X. Let G be such a child.

Suppose that w is not adjacent to a vertex in X. Let this vertex be x. As in the proof of Lemma 14, G can only be randomly traceable from w and x.

Suppose that $x = u$. Then the path $uvy_1x_1y_2x_2 \ldots y_{n-1}x_{n-1}y_n$ is not extendable to a hamiltonian path since it is not extendable to w. So G is not randomly traceable from x. Similarly, if $x = v$, then G is not randomly traceable from x.

Now suppose that x is distinct from u and v. Without loss of generality, let $x = x_{n-1}$. Then the path $x_{n-1}y_{n-1}x_{n-2}y_{n-2} \ldots x_1y_1uvy_n$ is not extendable to a hamiltonian path because it is not extendable to w. Therefore G is not randomly traceable from x.

Thus if w is not adjacent to every vertex in X, then G is infertile. So if G is fertile, then w is adjacent to every vertex in X. In this case, $G \cong K_{n+1,n+1} + uv$, where u and v lie in the same partite set. □

Characterization Theorem

Now this long list of lemmas can be used to prove a theorem characterizing fertile graphs.

Theorem 16. *If G is a fertile graph, then there exists $n \geq 1$ such that G is isomorphic to one of the following:*

P_n

C_n

$C_n + vx + vz$, *where $v \notin V(C_n)$ and xyz is an arc of C_n*

$C_n + vx + vy + vz$, where $v \notin V(C_n)$ and xyz is an arc of C_n
K_n
$K_n - uv$, where $u, v \in V(K_n)$
$K_{n,n}$
$K_{n,n} - uv$, where $u, v \in V(K_{n,n})$ are in different partite sets
$K_{n,n} + uv$, where $u, v \in V(K_{n,n})$ are in the same partite set
$K_{n,n+1}$
$K_{n,n+1} + uv$, where $u, v \in V(K_{n,n+1})$ are in the larger partite set

Proof. The theorem can be verified for graphs with fewer than six vertices by checking all possibilities.

Suppose that the theorem is true for graphs with fewer than n vertices, where $n \geq 6$. Let G be a graph with n vertices. Then G is the child of some fertile graph H, where H has $n-1$ vertices. So H is isomorphic to one of the graphs on the list. The lemmas of the previous section demonstrate that the fertile children of any graph on the list are also graphs on the list. Therefore G must also be a graph on this list.

By induction on n, the theorem is true. □

This list contains all graphs that are fertile. It only remains to find the graphs that are infertile. However, this is easy because of Theorem 8.

Theorem 17. *If G is infertile, then G can be formed by adding a new vertex v to a fertile graph H and attaching it to some of the vertices of H from which H is randomly traceable. Then, a path of any length (possibly 0) can be attached to v.*

Proof. Let G be infertile. G has a fertile ancestor because K_1 is a fertile ancestor of every graph that is randomly traceable from a vertex. So there exists a graph H such that H is a descendant of all other fertile ancestors of G. Thus there exists a sequence of graphs $G_1, G_2, \ldots, G_n$ such that $H = G_1$, $G = G_n$, and for all i such that $1 \leq i \leq n-1$, G_{i+1} is an infertile child of G_i.

G_2 can be formed by adding a new vertex v_2 and attaching it to some of the vertices of the fertile graph H. By assumption, G_2 is infertile at v_2. So G_3 is formed by adding a new vertex v_3 and attaching it to v_2. Continuing in this way, G_i is formed by adding a new vertex v_i and attaching it to v_{i-1}.

Eventually, G is formed by adding a new vertex v_2 to H and attaching it to some of the vertices of H. Then, the path $v_2 v_3 \ldots v_n$ is added (see Figure 2). □

Figure 2. An infertile graph with fertile ancestor C_4.

Thus Theorems 16 and 17 explicitly list all graphs that are randomly traceable from a vertex. Now a few corollaries can be drawn.

Corollary 18. *If G is randomly traceable from all of its vertices, then G is isomorphic to K_n, C_n, or $K_{n,n}$ for some $n \geq 1$.*

Proof. If G is randomly traceable from all of its vertices, then G must be fertile. So it is mentioned in Theorem 16. By checking each of these graphs, it can be verified that only K_n, C_n, and $K_{n,n}$ are randomly traceable from all of their vertices.□

This corollary was first proved by Chartrand and Kronk [ref1] using a different technique.

Corollary 19. *For all $n \geq 2$, there exist graphs with n vertices that are randomly traceable from exactly 1, 2, $\lfloor \frac{n+1}{2} \rfloor$, or n vertices. Moreover, these are the only numerical possibilities.*

Proof. All infertile graphs and K_1 are randomly traceable from exactly one vertex. P_n, $C_n + vx + vz$, $C_n + vx + vy + vz$, $K_n - uv$, and $K_{n,n} - uv$ are randomly traceable from exactly two of their vertices. $K_{n,n} + uv$ is randomly traceable from exactly n vertices. $K_{n,n+1}$ and $K_{n,n+1} + uv$ are randomly traceable from exactly $n+1$ vertices. K_n, C_n, and $K_{\frac{n}{2},\frac{n}{2}}$ are randomly traceable from exactly n vertices. □

Generalizations and Related Definitions

This section will discuss some variations on the basic concept of random traceability.

Definition 20. G is *randomly hamiltonian* from a vertex v if every path from v can be extended to a hamiltonian cycle.

This definition is a slight generalization of the definition of Chartrand and Kronk in [ref2].

Theorem 21. *If G is randomly hamiltonian from a vertex, then G is isomorphic to C_n, $K_{n,n}$, K_n, W_n (wheel), or $K_{n,n} + v$, where v is adjacent to every vertex of $K_{n,n}$.*

Proof. Suppose that G is randomly cycle-hamiltonian from a vertex v. Then every path starting at v can be extended to a hamiltonian cycle. However, a hamiltonian path can be formed from a hamiltonian cycle by removing a single edge. Therefore every path starting at v can be extended to a hamiltonian path. Hence, G is randomly traceable from v.

By checking each of the graphs mentioned in Theorems 16 and 17, it can be verified that the only possibilities are the graphs listed above. □

Theorem 22. *If G is randomly hamiltonian from all of its vertices, then G is isomorphic to C_n, $K_{n,n}$, or K_n.*

Proof. Let G be randomly cycle-hamiltonian. Then G is randomly cycle-hamiltonian from all of its vertices. So G must be mentioned in Theorem 21. The theorem can be proved by checking each of the graphs. □ This theorem

was first proved by Chartrand and Kronk [ref2] using a different technique.

Definition 23. If G is a graph and e is an edge of G, then G is *hamiltonian extendable* from e if every path containing e is contained in a hamiltonian path of G. If v is a vertex of G, then G is *hamiltonian extendable* from v if every path containing v is contained in a hamiltonian path of G.

In this situation, one is allowed to "extend" a path from both ends simultaneously rather than at only one end. Note that G is hamiltonian extendable from a vertex v if and only if G is hamiltonian extendable from every edge incident to v.

A related question is to consider graphs that are hamiltonian extendable from all of their edges. This is equivalent to considering graphs that are hamiltonian extendable from all of their vertices.

Definition 24. If u and v are vertices of a graph G, then u is a *guaranteed destination* of v if every path starting at v either contains u or can be extended to a path containing u.

Intuitively, this means if one starts at v, travels to any adjacent vertex, and repeats this process until it is impossible to continue, then it is guaranteed that u will be visited somewhere in this process. Note that any vertex is a guaranteed destination of itself.

Theorem 25. *G is randomly traceable from v if and only if for all $u \in V(G)$, u is a guaranteed destination of v.*

Proof. Suppose that G is randomly traceable from v. Let $u \in V(G)$. Let P be any path starting at v. Then P can be extended to a hamiltonian path Q. So Q contains u. Therefore u is a guaranteed destination of v.

Now suppose that for all $u \in V(G)$, u is a guaranteed destination of v. Let P be any path in G starting at v. Then P can be extended to a maximal path Q. Then Q must contain every vertex of G since every vertex is a guaranteed destination of v. Thus Q is a hamiltonian path. Hence G is randomly traceable from v. □

It can be seen that random traceability is essentially a special case of guaranteed destinations. A natural question is to consider graphs with a vertex u such that u is a guaranteed destination of every vertex. Theorem 25 suggests that this notion is a natural "dual" of random traceability.

These new definitions allow the opportunity for further research. Although randomly traceable graphs are now well understood, there is considerable room for further exploration in this topic.

Acknowledgement

The notion of random traceability from a vertex was inspired by a paper of Beineke, Goddard, and Hamburger [1] on random decomposability and by a paper of Chartrand and Nordhaus [3] on hamiltonian-connectedness.

References

[1] L. W. Beineke, W. D. Goddard, and P. Hamburger, Random Packings of Graphs, *Discrete Mathematics*, to appear.
[2] G. Chartrand and Kronk