MATH 0995 GROUP FINAL EXAM FALL 2004
SHOW ALL WORK. DO NOT USE A CALCULATOR.

1. (7 pts.) Simplify by adding (or subtracting) like terms wherever possible:
   \[
   \sqrt{x^2 - 1} - \frac{1}{3} + \frac{2}{3} - 3\sqrt{x^2 - 1} + 4 \cdot \frac{2}{3} - 3\sqrt{x^2 + 1}
   \]

2. (7 pts.) Simplify completely: \((8^{\frac{1}{3}} - 8^{\frac{1}{3}})^3\)

3. (7 pts.) Simplify completely: \(\left(\frac{2^{-1}}{2^{-1} + 3^{-1}}\right)^{-2}\)

4. (7 pts.) Solve: \(\frac{2x - 3}{6} = \frac{3x + 2}{4} + 1\)

5. (7 pts.) Solve for \(M\): \(\frac{2}{3M} - 1 = \frac{K}{L}\)

6. (7 pts.) Three numbers are such that the first number is 1 less than \(\frac{1}{2}\) of the second number. The third number is 2 more than \(\frac{1}{2}\) of the second number. The sum of the three numbers is 73. Find all three numbers.

7. (7 pts.) Simplify completely: \(\left(\frac{-2x^{-2}y^{-2}z^2}{x^{-4}y^3z^6}\right)^2\)

8. (6 pts.) Solve and graph the solution set: \(\frac{2(x - 3)}{-3} < \frac{4(x + 1)}{-3}\)

9. (7 pts.) The graph of a function, \(f\), is shown here.
   a) What is the domain of \(f\)?
   b) What is the range of \(f\)?
   c) What is \(f(0)\)?

10. (6 pts.) Let \(f\) be the function given by \(f(x) = \frac{2x}{\sqrt{3 - x}}\). What is the domain of \(f\)?

11. (7 pts.) Graph the function \(f(x) = -2\sqrt{x} + 5\). Label three points.

12. (7 pts.) Let \(g(x) = x + 2\). Find and simplify \(g(x - 1) + 3g(2x)\).
13. (6 pts.) Find the equation of the line that is perpendicular to the line \( y = 1 \) and passes through the point \((-1, 0)\).

14. (6 pts.) Find the equation of the line that is parallel to the line \( 5x - 7y = 8 \) and goes through the point \((2, -1)\).

15. (7 pts.) The area of a rectangular window is 84 square feet. The length is 5 feet longer than the width. Find the perimeter of the window.

16. (7 pts.) Solve:
\[
\begin{align*}
5x - 3y &= 9 \\
2x + 5y &= 16
\end{align*}
\]

17. (6 pts.) Factor completely: \( 24x^2 y^6 z^2 - 6x^2 y^2 z^4 \)

18. (6 pts.) Solve: \( x^4 + 10x^3 = -7x^3 \)

19. (7 pts.) Simplify:
\[
\frac{2x + 4}{2x^2 + 6x + 4}
\]

20. (7 pts.) Solve:
\[
\frac{x - 4}{2} + \frac{x + 2}{4} = \frac{15}{x - 3}
\]

21. (7 pts.) Sherita can ride her bike 12 miles in the same time that Carlos can ride 10 miles. If Carlos rides 1 mile/hour slower than Sherita, what is each person’s speed?

22. (6 pts.) Simplify completely: \( \sqrt{3xy^5} \cdot \sqrt{27x^3y^3} \)

23. (6 pts.) Simplify completely: \( \sqrt{32} + 3\sqrt{50} - \sqrt{18} \)

24. (7 pts.) Solve: \( x = 1 - \sqrt{x + 5} \)

25. (7 pts.) The diagonal of a square is \( 5\sqrt{2} \) inches. Find the length of the side of the square.

26. (6 pts.) Simplify completely: \( \left(\sqrt{2} - \sqrt{2i}\right)^2 \)

27. (7 pts.) Solve: \( (x - 1)(x - 3) = 4 \)

28. (7 pts.) Graph, labeling the vertex and all \( x \) and \( y \) intercepts: \( f(x) = 4 - x^2 \)

29. (7 pts.) Solve: \( x^2 + 2x + 1 > 0 \)

30. (6 pts.) Find: a) \( \log_{\sqrt{5}} \left(\sqrt{5}\right) \) b) \( \log_{100} (10) \) c) \( \log_3 (27) \)