MAT 1050 GROUP FINAL EXAM – FALL 2005

SHOW ALL WORK. DO NOT USE A CALCULATOR.

1. (7 pts.) Simplify by adding (or subtracting) like terms wherever possible:

\[-5 \cdot 3^3 + 3 \cdot 5^3 - \frac{1}{4} \sqrt{x - 2} - 3^3 + 15^3 + \frac{1}{3} \sqrt{x - 2}\]

2. (7 pts.) Simplify completely:

\[\left(2x^2y^{-1}\right)^{-2}\left(\frac{32x^6z^{-4}}{y^{-7}z}\right)\]

3. (7 pts.) Simplify completely:

\[\sqrt{6}\left(\sqrt{10} + \sqrt{5}\right) + \sqrt{5}\left(\sqrt{6} - \sqrt{27}\right)\]

4. (7 pts.) Simplify completely:

\[\left(\frac{4^{\frac{3}{2}}}{9^{\frac{1}{2}} + 4^{\frac{3}{2}}}\right)^{-2}\]

5. (6 pts.) Solve: \[-2 - \left|\frac{3x + 1}{2}\right| \leq -5\]

6. (6 pts.) Solve: \[|x - 3| = |x + 5|\]

7. (7 pts.) Three consecutive integers are such that the square of the first is 29 less than the product of the other two. What are the three consecutive integers?

8. (7 pts.) Solve for \(y\):

\[1 - \frac{xy - z}{x} = \frac{y}{xz}\]

9. (6 pts.) Let \(f\) be the function given by \(f(x) = \frac{x - 1}{2x^2 + 5x + 3}\).

What is the domain of \(f\)?

10. (6 pts.) Let \(g\) be the function given by \(g(x) = \sqrt{2x^2 + 1}\).

Find and simplify \([g(0) + g(-2)]^2\).

11. (7 pts.) Let \(f\) be the function given by \(f(x) = x^2 - 2x\).

Find and simplify \(\frac{f(a + h) - f(a - h)}{h}\).
12. (6 pts.) Find the equation of the line shown here:

![Graph of a line](image)

13. (6 pts.) Find the equation of the line that is perpendicular to the line $3y - 2x = 5$ and goes through the point $(0, 0)$.

14. (7 pts.) A support wire is attached to a flagpole to keep it vertical. The wire runs from the top of the pole to a point on the ground that is 8 yards from the base of the pole. The wire is 12 yards long. How tall is the pole?

15. (7 pts.) Solve, writing any non-real solutions in the form $a + bi$: $(x^2 - 4)(x^2 + 2x + 2) = 0$

16. (7 pts.) Graph, labeling the vertex and all $x$ and $y$ intercepts: $f(x) = -3x^2 + 6$

17. (7 pts.) Simplify completely:

$$\frac{3x + 13x - 4}{x + 2} + \frac{2x + 8}{x^2 - x - 6}$$

$$\frac{2}{x + 3} - \frac{2x + 8}{x^2 - 9}$$

18. (7 pts.) Solve: $3 - \sqrt{1 - 3x} = -x$

19. (7 pts.) Solve: $x^2 < 4$.

20. (7 pts.) Solve: $\frac{6}{4x + 7} \leq \frac{3}{2x - 1}$

21. (7 pts.) The graph of a function, $f$, is shown here.

a) What is the domain of $f$?

b) What is the range of $f$?

c) For what values of $x$ does $f(x) = 0$?
22. (7 pts.) Solve: \((3 - \sqrt{2x})^2 - (3 - \sqrt{2x}) - 12 = 0\)

23. (6 pts.) Find: a) \(\log_4 (4)\)   b) \(\log_{27} (3)\)   c) \(\log_5 (125)\)

24. (6 pts.) Using the approximate values \(\log_5 (3) = 0.7\) and \(\log_5 (12) = 1.5\) find:

   a) \(\log_5 (4)\)   b) \(\log_5 (\sqrt{15})\)   c) \(\log_5 (\frac{1}{9})\)

25. (7 pts.) Given \(f(x) = \log_2 (x) + \log_2 (3x - 1)\), find all inputs, \(x\), such that \(f(x) = 1\).

26. (7 pts.) Identify and sketch the curve given by \(x^2 + 6x + y^2 = -5\).

27. (7 pts.) Arrange the following numbers in order from smallest to largest:

   \(\log_2 (5)\)   \(\cos(-4)\)   \(\sin(-4)\)   \(\frac{\pi}{3}\)

28. (6 pts.) Convert to radians:

   a) \(\left(\frac{3\pi}{2}\right)^\circ\)

   b) \(-109^\circ\)

29. (6 pts.) For the right triangle shown here, find \(x\).

30. (7 pts.) Gabriel drives 40 miles at a certain speed. Hitting serious traffic, he is forced to drive at a speed that is 60 mph slower. He drives at this new speed for 50 miles. If the entire trip takes 3 hours, what is his speed during each part of the trip?