

Exercise set 1

Due Fri Sept. 11, 2009.

1. Let G and H be groups regarded as categories with one object.
 - (a) what is a functor $F: G \rightarrow H$?
 - (b) what is a natural transformation $\eta: F_1 \rightarrow F_2: G \rightarrow H$?

2. Show that the following are equivalent:
 - (a) X is contractible (i.e. $1_X \simeq$ a constant map)
 - (b) $[X, Y]$ has one element for path connected Y
 - (c) $[Y, X]$ has one element for all Y
 - (d) $X \simeq *$
 - (e) $X \simeq Y$ if Y is contractible.
 - (f) X is a retract of CX . [$CX = X \times I / X \times 0$]

3. Let $\alpha, \beta, \gamma: (S^1, 1) \rightarrow (X, x_0)$ satisfy $\alpha\beta \simeq \alpha\gamma$ rel $\{1\}$.
Show that $\beta \simeq \gamma$ rel $\{1\}$.