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Math 7510, Fall 2009, Homework 2
due 18 September 2009

The following are transcribed from Greenberg and Harper(G-H), but shortened, to save typing, and corrected, to make them more interesting.

3.7 Let X be 0-connected. Then TFAE:

1. X is 1-connected.
2. Every map of S^1 into X extends to a map of the closed unit disk D^2 into X. (D^2 is my name for G-H's E^2.)
3. If σ and τ are paths in X with the same initial points and the same terminal points then $\sigma \simeq \tau$ rel $\{0, 1\}$.

3.8 Let $CX = X \times I/X \times 0$ be the cone on X as before. Embed $X \longrightarrow CX$ by $x \mapsto (x, 1)$. Generalize 3.7(2) to show $X \longrightarrow Y$ is homotopically trivial iff f extends to $CX \longrightarrow Y$.

3.10 Let $f, g : X \longrightarrow S^n$ be maps such that for all $x \in X$, $f(x)$ and $g(x)$ are not antipodal. Show that $f \simeq g$. If in addition there is an $x_0 \in X$ such that $f(x_0) = g(x_0)$, then $f \simeq g$ rel x_0.

3.12 Let X be 0-connected and suppose that every $S^1 \longrightarrow X$ is homotopically trivial, but not necessarily by a homotopy which fixes the basepoint. Show that $\pi_1(X, x_0) = 1$.