R. Bruner
Math 7510, Fall 2009, Homework 3
due 25 September 2009

1. (4.12 in Greenberg and Harper) Let $X = U \cup V$, where U and V are open sets satisfying

   - $x_0 \in U \cap V$ and $\pi_0(U \cap V, x_0) = *$, and
   - $\pi_1(U, x_0) = \pi_1(V, x_0) = 1$.

   Show that $\pi_1(U \cup V, x_0) = 1$.

2. (4.13 in Greenberg and Harper) Show that S^n is simply connected if $n > 1$.

3. (5.10 in Greenberg and Harper) Suppose E is connected and locally path connected. Let the group G act *properly discontinuously* on E by homeomorphisms. This means that for each $e \in E$ there exists an open neighborhood V such that $V \cap gV = \emptyset$ if $g \neq 1$. Let $E/G = \{Ge \mid e \in E\}$ be the space of orbits and $p : E \longrightarrow E/G$ the projection, $p(e) = Ge$.

   Show that p is a covering, G is the group of covering transformations, and that $p_* \pi_1(E, e_0)$ is normal in $\pi_1(E/G, Ge_0)$.

4. Let $\mathbf{Z}$ act on $\mathbf{R}^2 - \{0\}$ by

$$n \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2^n x \\ 2^{-n} y \end{pmatrix}$$

   Show that this action is *not* properly discontinuous.