R. Bruner

Math 7510, Fall 2009, Homework 8

due 13 November 2009

1. Compute the homology of the 'stack of two tori', $X = X_1 \cup X_2$, where X_1 and X_2 are the tori $(r-2)^2 + z^2 = 1$ and $(r-2)^2 + (z-2)^2 = 1$ in cylindrical coordinates.

2. Compute the homology of $\mathrm{Cof}(S^1 \times S^1 \xrightarrow{\mu} S^1)$, where μ is the multiplication $\mu(z_1, z_2) = z_1 z_2$, viewing S^1 as the unit complex numbers.

3. Suppose that

$$\begin{array}{ccc} P & \xrightarrow{i_1} & A_1 \\ \downarrow{\scriptstyle i_2} & & \downarrow{\scriptstyle p_1} \\ A_2 & \xrightarrow{p_2} & B \end{array}$$

is a pullback in the category of R-modules. Show that

(a) the natural map $\mathrm{Ker}\, i_2 \longrightarrow \mathrm{Ker}\, p_1$ is an isomorphism, and

(b) if p_1 is onto, then i_2 is onto.

4. Let β_p be the Bockstein associated to the coefficient sequence

$$0 \longrightarrow \mathbf{Z}/p\mathbf{Z} \longrightarrow \mathbf{Z}/p^2\mathbf{Z} \longrightarrow \mathbf{Z}/p\mathbf{Z} \longrightarrow 0$$

and let $\bar{\beta}_p$ be the Bockstein associated to the coefficient sequence

$$0 \longrightarrow \mathbf{Z} \longrightarrow \mathbf{Z} \longrightarrow \mathbf{Z}/p\mathbf{Z} \longrightarrow 0.$$

(a) Show that $\beta_p = q_* \bar{\beta}_p$, where q_* is the homomorphism induced by the quotient map $q : \mathbf{Z} \longrightarrow \mathbf{Z}/p\mathbf{Z}$.

(b) Show that $\beta_p \beta_p = 0$.