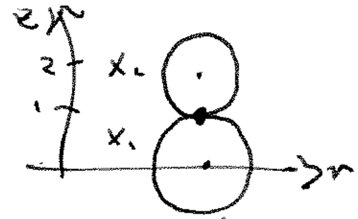
HW 8



cross-section

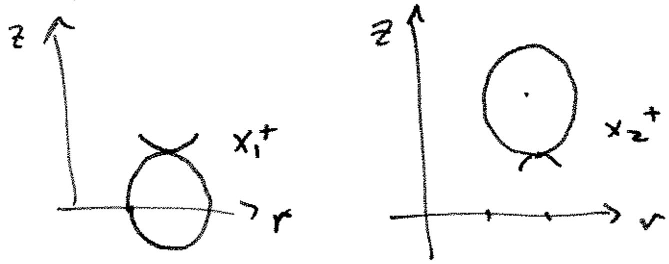
1. $X_1 \cap X_2$ is the circle $r=2,\ z=1$ since

X_1 is $(r-2)^2+z^2=1$ and X_2 is $(r-2)^2+(z-2)^2=1$

"Fatten them up" by adding to X_1 the set

$X_2 \cap \{z<1+\varepsilon\}$ and to X_2 the set $X_1 \cap \{z>1-\varepsilon\}$

Fattened versions in cross-section



Now $\overset{\circ}{X_1^+} \cup \overset{\circ}{X_2^+} = X_1 \cup X_2$,

	$H_i(X_1\cap X_2)$		$H_i(X_1)\oplus H_i(X_2)$		$H_i(X_1\cup X_2)$
0	$\mathbb{Z}$	$\xrightarrow{\begin{pmatrix}1\\-1\end{pmatrix}}$	$\mathbb{Z}^2$	$\longrightarrow$	$\mathbb{Z}$
1	$\mathbb{Z}$	$\xrightarrow{\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}}$	$\mathbb{Z}^4$	$\longrightarrow$	$\mathbb{Z}^3$
2	0		$\mathbb{Z}^2$	$\longrightarrow$	$\mathbb{Z}^2$

mono, so $\partial = 0$ and onto

6

2.

$$\begin{tikzcd} 1\times S^1 \arrow[d] \arrow[dr, equal] & \\ S^1\times S^1 \arrow[r, "\mu"] & S^1 \\ S^1\times 1 \arrow[u] \arrow[ur, equal] & \end{tikzcd} \Rightarrow \begin{tikzcd} z_1 \arrow[d, mapsto] & \\ 1\otimes z_1 \arrow[r, mapsto, "\mu_*"] & z_1 \\ z_1\otimes 1 \arrow[ur, mapsto] & \\ z_1 \arrow[u, mapsto] & \end{tikzcd}$$

So $\mu_* = (1,1)$ in H_1

Reduced

$\tilde{H}_*$	$(S^1\times S^1$		$\to S^1$	$\to \mathrm{Cof}(\mu))$
0	0	$\longrightarrow$	0	0
1	$\mathbb{Z}^2$	$\xrightarrow{(1\ 1)}$	$\mathbb{Z}$	0
2	$\mathbb{Z}$	$\longrightarrow$	0	$\mathbb{Z}$
3	0		0	$\mathbb{Z}$

$\begin{pmatrix}1\\-1\end{pmatrix}$ $\cong$

given homology, Calc. μ_*

Unreduced, $H_*\,\mathrm{Cof}\,\mu = \mathbb{Z}\ \ 0\ \ \mathbb{Z}\ \ \mathbb{Z}$.

2 + 4

6 = →

3.

$$\begin{tikzcd} P \arrow[r, "i_1"] \arrow[d, "i_2"'] & A_1 \arrow[d, "p_1"] \\ A_2 \arrow[r, "p_2"'] & B \end{tikzcd}$$

so $P = \left\{ \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \in A_1 \times A_2 \;\middle|\; p_1(a_1) = p_2(a_2) \right\}$

$$\begin{tikzcd} & P \arrow[dl, "i_1"'] \arrow[dr, "i_2"] & \\ a_1 & & a_2 \end{tikzcd}$$

pullback in R-Mod

(a) $\operatorname{Ker} i_2 \to \operatorname{Ker} p_1$ is iso:

Pf: $\operatorname{Ker} i_2 = \left\{ \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \middle| p_1 a_1 = p_2 a_2 \text{ and } a_2 = 0 \right\} = \left\{ \begin{pmatrix} a_1 \\ 0 \end{pmatrix} \middle| p_1 a_1 = 0 \right\} \cong \{ a_1 \in A_1 \mid p_1 a_1 = 0 \}$

2 $= \operatorname{Ker} p_1$ // and map is $\begin{pmatrix} a_1 \\ 0 \end{pmatrix} \mapsto a_1$

(b) p_1 onto $\Rightarrow$ i_2 onto

2 Pf: If $a_2 \in A_2$ choose a_1 s.t. $p_1(a_1) = p_2(a_2)$. Then $i_2 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = a_2$. //

4.

$$\begin{tikzcd} 0 \arrow[r] & \mathbb{Z} \arrow[r, "p"] \arrow[d, "q"] & \mathbb{Z} \arrow[r, "q"] \arrow[d, "q"] & \mathbb{Z}/p\mathbb{Z} \arrow[r] \arrow[d, equal] & 0 \\ 0 \arrow[r] & \mathbb{Z}/p \arrow[r] & \mathbb{Z}/p^2 \arrow[r] & \mathbb{Z}/p\mathbb{Z} \arrow[r] & 0 \end{tikzcd}$$

gives $H_i(X; \mathbb{Z}/p) \xrightarrow{\bar{\beta}_p} H_{i-1}(X; \mathbb{Z}) \xrightarrow{\rho_*} H_{i-1}(X; \mathbb{Z}$

gives a map of long exact sequences

$$\begin{tikzcd} \cdots H_{i+1}(X; \mathbb{Z}/p) \arrow[r, "\bar{\beta}_p"] \arrow[d, equal] & H_i(X; \mathbb{Z}) \arrow[r, "p"] \arrow[d, "q_*"] & H_i(X; \mathbb{Z}) \arrow[r, "q_*"] \arrow[d, "\bar{q}_*"] & H_i(X; \mathbb{Z}/p) \arrow[r, "\bar{\beta}_p"] \arrow[d, equal] & \cdots \\ H_{i+1}(X; \mathbb{Z}/p) \arrow[r, "\beta_p"'] & H_i(X; \mathbb{Z}/p) \arrow[r] & H_i(X; \mathbb{Z}/p^2) \arrow[r] & H_i(X; \mathbb{Z}/p) \arrow[r, "\beta_p"'] & \cdots \end{tikzcd}$$

(a) By naturality, $\beta_p = q_* \bar{\beta}_p$

3 (b) $\beta_p \beta_p = q_* \bar{\beta}_p q_* \bar{\beta}_p$ and $\bar{\beta}_p q_* = 0$ so $\beta_p \beta_p = 0$. //