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Math 7510, Fall 2009, Homework 9
due 23 November 2009

1. Suppose that $\operatorname{hd} R \le 1$, that A is a flat R-module and that B is a submodule of A. Show that B is also flat.

2. Suppose that A_i, B_i, and C_i are chain complexes, for $i \in \{1, 2, 3\}$, and that the diagram below has exact rows and columns.

$$\begin{array}{ccccccccc} & & 0 & & 0 & & 0 & & \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & A_1 & \longrightarrow & A_2 & \longrightarrow & A_3 & \longrightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & B_1 & \longrightarrow & B_2 & \longrightarrow & B_3 & \longrightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & C_1 & \longrightarrow & C_2 & \longrightarrow & C_3 & \longrightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ & & 0 & & 0 & & 0 & & \end{array}$$

Let ∂^h and ∂^v denote the 'horizontal' and 'vertical' boundary maps, respectively. Show that they anti-commute (i.e., commute in the graded sense) $\partial^h \partial^v = -\partial^v \partial^h$.

$$\begin{array}{ccc} H_{n+1}(C_3) & \xrightarrow{\quad \partial^h \quad} & H_n(C_1) \\ \partial^v \downarrow & (-1) & \downarrow \partial^v \\ H_n(A_3) & \xrightarrow[\quad \partial^h \quad]{} & H_{n-1}(A_1) \end{array}$$

3. Compute, using coefficients in $\mathbf{Z}$, $\mathbf{Z}[1/2]$ and $\mathbf{Z}/2\mathbf{Z}$:

   (a) $H_*(S^p \times S^q)$

   (b) $H_*(RP^2 \times S^3)$

   (c) $H_*(S^2 \times RP^3)$