Abstract. To make it as easy for the user as possible, we want to be able to specify a module over the Steenrod algebra by simply giving the $Sq^{2i}$. Here, we point out a relation in the Steenrod algebra which gives an efficient way to compute all the $Sq^i$ given only the $Sq^{2i}$.

1. Introduction

Let $A$ be the mod 2 Steenrod algebra. Generalizing the relation $Sq^1 Sq^{2n} = Sq^{2n+1}$, we have the following relation in $A$.

Proposition 1.

$$Sq^{2a+1,n+2^a} = Sq^{2a} Sq^{2a+1,n} + \sum_{k=0}^{a-1} Sq^{2a+1,n+2^a-2^k} Sq^{2k}$$

Proof. Expand $Sq^{2a} Sq^{2a+1,n}$ by the Adem relations. The only terms which are nonzero are those occurring in the formula, by an elementary exercise in binomial coefficients. Specifically, the bits in the binary expansion of $2a - 2j$ are all contained within the bits of the binary expansion of $2a+1,n - j - 1$ iff either $j = 0$ or $j = 2^k$ with $0 \leq k \leq a - 1$.

Remark 2. Therefore, we can compute $Sq^i$ inductively by writing $i = 2^a + 2^{a+1}n$ and using the calculation in lower degrees inductively, together with the given action of the $Sq^{2k}$.

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