1. Answer True or False, where matrices $A$, $B$, and $C$ have appropriate size.
   a) If $AC = BC$, then $A = B$.
   b) If $AB = O$, where $O$ is a zero matrix, then $A = O$ or $B = O$.
   c) If $A^2 = I$, where $I$ is the identity matrix, then $A = \pm I$.

2. Determine which sets are sub-spaces and explain why.
   a) $\{[x_1, 2x, 3x] : x \in \mathbb{R}\}$ in $\mathbb{R}^3$.
   b) $\{[x, x - 1] : x \in \mathbb{R}\}$ in $\mathbb{R}^2$.

3. Suppose that $\mathbf{u}$ and $\mathbf{v}$ are vectors such that $\|\mathbf{u}\| = 2$, $\|\mathbf{v}\| = 2$, $(\mathbf{u}, \mathbf{v}) = 2$.
   a) Evaluate $\|\mathbf{u} + 2\mathbf{v}\|$ and $\|\mathbf{u} - 2\mathbf{v}\|$.
   b) Find the angle between $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.
   c) Find the angle between $\mathbf{u}$ and $\mathbf{u} - \mathbf{v}$.

4. Find a basis for a) the row space; and b) the nullspace of
   \[
   \begin{pmatrix}
   1 & 3 & 5 & 7 \\
   2 & 0 & 4 & 2 \\
   3 & 2 & 8 & 7
   \end{pmatrix}
   \]

   What is the dimension for each space?

5. Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be linearly independent vectors.
   a) Determine whether $\mathbf{u} - \mathbf{v}$, $\mathbf{v} - \mathbf{w}$ and $\mathbf{w} - \mathbf{u}$ are linearly independent. Why?
   b)* (extra credit) Determine whether $\mathbf{u} + \mathbf{v}$, $\mathbf{v} + \mathbf{w}$ and $\mathbf{w} + \mathbf{u}$ are linearly independent.

Note: Show each step for partial credit.
1. All false.

2. a) Yes (proof is skipped); b) No since \((0, -1) + (0, -1) = (0, -2)\) is not in the set.

3. a) Direct calculation gives,
\[
\|u \pm v\|^2 = \|u\|^2 + 2(u, v) + \|v\|^2 = 4 + 4 + 4.
\]
Therefore,
\[
\|u + v\| = 2\sqrt{3}, \quad \|u - v\| = 2.
\]
Since
\[
(u + v) \cdot (u - v) = \|u\|^2 - \|v\|^2 = 0,
\]
therefore, the angle is \(\pi/2\).

c) Since
\[
(u, u - v) = \|u\|^2 - (u, v) = 2, \quad \frac{(u, u - v)}{\|u\|\|u - v\|} = \frac{1}{2},
\]
therefore, the angle is \(\pi/3\).

4.
\[
\begin{pmatrix}
1 & 3 & 5 & 7 \\
2 & 0 & 4 & 2 \\
3 & 2 & 8 & 7 \\
\end{pmatrix}
\implies
\begin{pmatrix}
1 & 3 & 5 & 7 \\
0 & 1 & 1 & 2 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

A basis for the row space \((1, 3, 5, 7), (0, 1, 1, 2)\);
A basis for the null space \((2, 1, -1, 0)^T, (1, 2, 0, -1)^T\).
All dimensions are 2.

5. a) They are linearly dependent since
\[
(u - v) + (v - w) + (w - u) = 0.
\]

b) They are linearly independent since from
\[
c_1(u + v) + c_2(v + w) + c_3(w + u) = 0
\]
we obtain
\[
(c_3 + c_1)u + (c_1 + c_2)v + (c_2 + c_3)w = 0.
\]
Recall that \(u, v,\) and \(w\) are linearly independent, therefore,
\[
c_3 + c_1 = 0, \quad c_1 + c_2 = 0, \quad c_2 + c_3 = 0,
\]
which results in \(c_1 = c_2 = c_3 = 0.\)