TEST 1

MAT 2250, Section 001, Winter 2005

NAME: **SSN**:

1. Answer True or False, where matrices A, B, and C have appropriate size.
a) If $AC = BC$, then $A = B$.
b) If $AB = O$, where O is a zero matrix, then $A = O$ or $B = O$.
c) If $A^2 = I$, where I is the identity matrix, then $A = \pm I$.

2. Determine which sets are sub-spaces and explain why.
a) $\{[x_1, 2x, 3x] : \ x \in R\}$ in R^3.
b) $\{[x, x-1] : \ x \in R\}$ in R^2.

3. Suppose that $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors such that $\|\boldsymbol{u}\| = 2$, $\|\boldsymbol{v}\| = 2$, $(\boldsymbol{u}, \boldsymbol{v}) = 2$.
a) Evaluate $\|\boldsymbol{u} + 2\boldsymbol{v}\|$ and $\|\boldsymbol{u} - 2\boldsymbol{v}\|$.
b) Find the angle between $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$.
c) Find the angle between $\boldsymbol{u}$ and $\boldsymbol{u} - \boldsymbol{v}$.

4. Find a basis for a) the row space; and b) the nullspace of

$$\begin{pmatrix} 1 & 3 & 5 & 7 \\ 2 & 0 & 4 & 2 \\ 3 & 2 & 8 & 7 \end{pmatrix}$$

What is the dimension for each space?

5. Let $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w}$ be linearly independent vectors.
a) Determine whether $\boldsymbol{u} - \boldsymbol{v}$, $\boldsymbol{v} - \boldsymbol{w}$ and $\boldsymbol{w} - \boldsymbol{u}$ are linearly independent. Why?
b)* (extra credit) Determine whether $\boldsymbol{u} + \boldsymbol{v}$, $\boldsymbol{v} + \boldsymbol{w}$ and $\boldsymbol{w} + \boldsymbol{u}$ are linearly independent.

Note: Show each step for partial credit.

1. All false.

2. a) Yes (proof is skipped); b) No since $(0,-1)+(0,-1)=(0,-2)$ is not in the set.

3. a) Direct calculation gives,

$$\|\boldsymbol{u}\pm\boldsymbol{v}\|^2=\|\boldsymbol{u}\|^2+2\langle\boldsymbol{u},\boldsymbol{v}\rangle+\|\boldsymbol{v}\|^2=4\pm4+4.$$

Therefore,

$$\|\boldsymbol{u}+\boldsymbol{v}\|=2\sqrt{3},\quad \|\boldsymbol{u}-\boldsymbol{v}\|=2.$$

Since

$$(\boldsymbol{u}+\boldsymbol{v})\cdot(\boldsymbol{u}-\boldsymbol{v})=\|\boldsymbol{u}\|^2-\|\boldsymbol{v}\|^2=0,$$

therefore, the angle is $\pi/2$.

c) Since

$$(\boldsymbol{u},\boldsymbol{u}-\boldsymbol{v})=\|\boldsymbol{u}\|^2-(\boldsymbol{u},\boldsymbol{v})=2,\quad \frac{(\boldsymbol{u},\boldsymbol{u}-\boldsymbol{v})}{\|\boldsymbol{u}\|\|\boldsymbol{u}-\boldsymbol{v}\|}=\frac{1}{2},$$

therefore, the angle is $\pi/3$.

4.

$$\begin{pmatrix}1&3&5&7\\2&0&4&2\\3&2&8&7\end{pmatrix}\Longrightarrow\begin{pmatrix}1&3&5&7\\0&1&1&2\\0&0&0&0\end{pmatrix}$$

A basis for the row space $(1,3,5,7),(0,1,1,2)$;
A basis for the null space $(2,1,-1,0)^T,(1,2,0,-1)^T$.
All dimenssions are 2.

5. a) They are linearly dependent since

$$(\boldsymbol{u}-\boldsymbol{v})+(\boldsymbol{v}-\boldsymbol{w})+(\boldsymbol{w}-\boldsymbol{u})=\mathbf{0}.$$

b) They are linearly independent since from

$$c_1(\boldsymbol{u}+\boldsymbol{v})+c_2(\boldsymbol{v}+\boldsymbol{w})+c_3(\boldsymbol{w}+\boldsymbol{u})=\mathbf{0}$$

we obtain

$$(c_3+c_1)\boldsymbol{u}+(c_1+c_2)\boldsymbol{v}+(c_2+c_3)\boldsymbol{w}=\mathbf{0}.$$

Recall that $\boldsymbol{u},\boldsymbol{v}$, and $\boldsymbol{w}$ are linearly independent, therefore,

$$c_3+c_1=0,\quad c_1+c_2=0,\quad c_2+c_3=0,$$

which results in $c_1=c_2=c_3=0$.