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Optimal Superconvergence of Energy Conserving Local Discontinuous Galerkin Methods for Wave Equations

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Abstract. This paper is concerned with numerical solutions of the LDG method for 1D wave equations. Superconvergence and energy conserving properties have been studied. We first study the superconvergence phenomenon for linear problems when alternating fluxes are used. We prove that, under some proper initial discretization, the numerical trace of the LDG approximation at nodes, as well as the cell average, converge with an order $2k+1$. In addition, we establish $k+2$-th order and $k+1$-th order superconvergence rates for the function value error and the derivative error at Radau points, respectively. As a byproduct, we prove that the LDG solution is superconvergent with an order $k+2$ towards the Radau projection of the exact solution. Numerical experiments demonstrate that in most cases, our error estimates are optimal, i.e., the error bounds are sharp. In the second part, we propose a fully discrete numerical scheme that conserves the discrete energy. Due to the energy conserving property, after long time integration, our method still stays accurate when applied to nonlinear Klein-Gordon and Sine-Gordon equations.

AMS subject classifications: 65L20, 65M12, 65N12

Key words: Local discontinuous Galerkin methods (LDG), wave equations, superconvergence, energy conserving.

1 Introduction

We study the local discontinuous Galerkin (LDG) method for the following 1D wave equations

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