ESAIM: M2AN 51 (2017) 467-486
DOI: 10.1051/m2an/2016026

ESAIM: Mathematical Modelling and Numerical Analysis
www.esaim-m2an.org

SUPERCONVERGENCE OF DISCONTINUOUS GALERKIN METHODS BASED ON UPWIND-BIASED FLUXES FOR 1D LINEAR HYPERBOLIC EQUATIONS *

Waixiang Cao[1], Dongfang Li[2], Yang Yang[3] and Zhimin Zhang[1,4]

Abstract. In this paper, we study superconvergence properties of the discontinuous Galerkin method using upwind-biased numerical fluxes for one-dimensional linear hyperbolic equations. A $(2k+1)$th order superconvergence rate of the DG approximation at the numerical fluxes and for the cell average is obtained under quasi-uniform meshes and some suitable initial discretization, when piecewise polynomials of degree k are used. Furthermore, surprisingly, we find that the derivative and function value approximation of the DG solution are superconvergent at a class of special points, with an order $k+1$ and $k+2$, respectively. These superconvergent points can be regarded as the generalized Radau points. All theoretical findings are confirmed by numerical experiments.

Mathematics Subject Classification. 65M15, 65M60, 65N30.

Received January 20, 2016. Revised March 24, 2016. Accepted April 21, 2016.

1. Introduction

In this paper, we study and analyze the discontinuous Galerkin (DG) method for the following one-dimensional linear hyperbolic conservation laws

$$\begin{aligned} &u_t + u_x = 0, && (x,t) \in [0, 2\pi] \times (0, T], \\ &u(x,0) = u_0(x), && x \in R, \end{aligned} \tag{1.1}$$

where u_0 is sufficiently smooth. We will consider both the periodic boundary condition $u(0,t) = u(2\pi,t)$ and the Dirichlet boundary condition $u(0,t) = g(t)$.

Keywords and phrases. Discontinuous Galerkin methods, superconvergence, generalized Radau points, upwind-biased fluxes.

* *This work is supported in part by the China Postdoctoral Science Foundation 2015M570026, and the National Natural Science Foundation of China (NSFC) under Grants Nos. 91430216, 11471031, 11501026, and the US National Science Foundation (NSF) through grant DMS-1419040.*

[1] Beijing Computational Science Research Center, Zhongguancun Software Park II, No. 10 West Dongbeiwang Road, Haidian District, Beijing 100094, P.R. China.

[2] School of Mathematics and Statistics, Huazhong University of Science and Technology, 1037 Luoyu Rd, Hongshan, Wuhan, Hubei 430074, P.R. China.

[3] Department of Mathematical Sciences, Michigan Technological University, 1400 Townsend Drive, Houghton, MI 49931, USA. yyang7@mtu.edu

[4] Department of Mathematics, Wayne State University, 42 W. Warren Ave. Detroit, MI 48202, USA.

Article published by EDP Sciences