Superconvergence of the Direct Discontinuous Galerkin Method for Convection-Diffusion Equations

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This paper is concerned with superconvergence properties of the direct discontinuous Galerkin (DDG) method for one-dimensional linear convection-diffusion equations. We prove, under some suitable choice of numerical fluxes and initial discretization, a $2k$-th and $(k+2)$ -th order superconvergence rate of the DDG approximation at nodes and Lobatto points, respectively, and a $(k+1)$ -th order of the derivative approximation at Gauss points, where k is the polynomial degree. Moreover, we also prove that the DDG solution is superconvergent with an order $k+2$ to a particular projection of the exact solution. Numerical experiments are presented to validate the theoretical results. Numer Methods Partial Differential Eq 33: 290–317, 2017

Keywords: convection-diffusion equations; direct discontinuous Galerkin methods; superconvergence

I. INTRODUCTION

In this paper, we study the superconvergence of the direct discontinuous Galerkin (DDG) method for the one-dimensional linear convection-diffusion equation

$$\begin{aligned} &\partial_t u + \partial_x f(u) = \partial_x^2 u, \quad (x,t) \in [a,b] \times [0,T], \\ &u(x,0) = u_0(x), \quad x \in [a,b], \end{aligned} \tag{1.1}$$

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