Superconvergence of the Direct Discontinuous Galerkin Method for Convection-Diffusion Equations

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This paper is concerned with superconvergence properties of the direct discontinuous Galerkin (DDG) method for one-dimensional linear convection-diffusion equations. We prove, under some suitable choice of numerical fluxes and initial discretization, a $2k$-th and $(k + 2)$-th order superconvergence rate of the DDG approximation at nodes and Lobatto points, respectively, and a $(k + 1)$-th order of the derivative approximation at Gauss points, where $k$ is the polynomial degree. Moreover, we also prove that the DDG solution is superconvergent with an order $k + 2$ to a particular projection of the exact solution. Numerical experiments are presented to validate the theoretical results.


Keywords: convection-diffusion equations; direct discontinuous Galerkin methods; superconvergence

I. INTRODUCTION

In this paper, we study the superconvergence of the direct discontinuous Galerkin (DDG) method for the one-dimensional linear convection-diffusion equation

$$\partial_t u + \partial_x f(u) = \partial_x^2 u, \quad (x, t) \in [a, b] \times [0, T],$$

$$u(x, 0) = u_0(x), \quad x \in [a, b],$$

(1.1)