Recent mathematical studies of diffractive and nano optics

Gang Bao, Michigan State University

Friday, April 17, 2:00–2:50, Room 1525, Law School

The recent enabling technologies of high-performance computing facilities and microlithographic fabrication techniques have led to applications of diffraction in optics from subwavelength structures, establishing diffractive optics and nano optics as two of the most rapidly advancing areas of current research in optical engineering. The substantial growth of significant applications of diffractive and nano optics has driven the need for novel mathematical models and numerical algorithms. Accurate modeling of electromagnetic fields within these materials presents challenging and significant mathematical questions both in theory and computation.

The speaker will discuss some recent developments and challenges including three dimensional electromagnetic wave propagation in periodic chiral or nonlinear structures; global uniqueness and numerical solution of the ill-posed inverse diffractive problems; numerical techniques for solving linear and nonlinear Maxwell’s equations; nonlinear second harmonic generation in dielectric and metallic nonlinear media; and multiscale modeling, analysis, and computation of optical responses of nano structures.

Radial basis function spectral methods and their application to flow on a sphere and nonlinear wave equations

John P. Boyd, University of Michigan, Ann Arbor

Friday, April 17, 2:50–3:40, Room 1525, Law School

Radial basis functions (RBF) have become popular in computer graphics and scattered data interpolation; in recent years, they have also been employed to solve partial differential equations. We shall describe the virtues and faults of RBF spectral methods. One illustration is an RBF model for solving the barotropic vorticity equation on the sphere. Another is the Benjamin-Ono equation (with its nonlocal, integral operator) and its generalizations.
The hybridizable discontinuous Galerkin methods
Bernardo Cockburn, University of Minnesota
Saturday, April 18, 8:30–9:20, Room 1525, Law School

The success of the application of discontinuous Galerkin methods to nonlinear hyperbolic problems in the 90’s fueled the recent exploration of new and old DG methods for elliptic problems. Although the DG methods are clearly ideal for adaptive strategies, the method has been criticized, especially within the structural mechanics community, for having significantly more degrees of freedom than the continuous Galerkin method (for the same mesh), and for producing less accurate solutions than certain mixed methods. The hybridizable discontinuous Galerkin methods appeared as a response to this criticism.

In this talk, we introduce these methods in the framework of second-order elliptic problems, show that they can be efficiently implemented, and argue that they are actually more accurate than all previously known discontinuous Galerkin methods. Numerical comparisons with the continuous and with some mixed methods will be presented.

Lagrangian particle methods
Robert Krasny, University of Michigan, Ann Arbor
Saturday, April 18, 9:20–10:10, Room 1525, Law School

Particle methods are motivated by mathematical and physical considerations. On the mathematical side they arise from looking at the Green’s function of the given problem, and on the physical side they arise from looking at how point masses, point charges, or point vortices interact. In a time-dependent problem, the Lagrangian approach keeps track of the flow map and this is useful to gain insight into complex dynamics. Special techniques have been developed for these methods including kernel smoothing for stability, adaptive interpolation for accuracy, and treecode algorithms for efficiency. This talk will describe some recent developments in this area. In particular, we discuss treecode algorithms for multiquadric radial basis functions and screened Coulomb interactions, and simulations of 1D collisionless plasmas and 3D vortex sheet flow.
Efficient spectral methods for high-dimensional PDEs
Jie Shen, Purdue University, West Lafayette
Saturday, April 18, 2:00–2:50, Room 1525, Law School

Many scientific, engineering and financial applications require solving high-dimensional PDEs. However, traditional tensor product based algorithms suffer from the so called "curse of dimensionality". We shall present a new spectral-Galerkin method for non-periodic problems and/or in the whole space. The method is based on two basic ingredients: (i) Choosing the frequencies of the trial functions from the "hyperbolic cross"; (ii) Using a sparse grid or a lattice rule to perform the numerical interpolation/integration. We will present rigorous estimates as well as efficient numerical algorithms for elliptic equations. We will also present some preliminary numerical results for the six-dimensional BGK model.

Models and numerical solutions for switching diffusions
George Yin, Wayne State University
Saturday, April 18, 2:50–3:40, Room 1525, Law School

In this talk, we present some of our recent work on switching diffusions, which involve both continuous dynamics and discrete events. In our formulation, in contrast to the Markovian regime-switching diffusions, the pure jump component depends on the continuous component. We design numerical methods for solving the corresponding stochastic differential equations with switching, and study the associated numerical solution for stochastic control problems.