(Due at the beginning of class on Thursday May 17, 2007)
Hand in solutions to the following:
1. p. 101 problem 10
2. p. 101 problem 13
3. Let \( G \) denote the set of non-zero real numbers. For \( a \) and \( b \) in \( G \), define
   \[
   a * b = \begin{cases} 
   ab & \text{if } a > 0, \\
   a/b & \text{if } a < 0.
   \end{cases}
   \]
   (a) Prove: \((G, *)\) is a group.
   (b) Is \((G, *)\) abelian? Prove your answer.

Also do, but do not hand in,
p.100–102 problems 1,2,11,12,14,15,22,23.

MAT 5420 S/S ’07 Assignment # 2

(Due at the beginning of class on Thursday May 24, 2007)
Read Appendix A.4, pp. 102–108, and §§1.1,1.3,1.4.
Hand in solutions to the following:
1. p. 113 problem 6a
2. p. 114 problem 19a,b
3. Let \((G, *)\) be the group of Problem 3, Assignment 1. Let
   \[
   J = \{3^n \mid n \in \mathbb{Z}\} \cup \{-3^n \mid n \in \mathbb{Z}\}.
   \]
   Prove: \(J\) is a subgroup of \(G\) under \(*\).

Also do, but do not hand in,
1. p. 113–114 problems 6b,7–9,13,17,18
2. Let \((G, *)\) be the group of Problem 3, Assignment 1. Let \(H = \{r \in \mathbb{R} \mid r > 0\}\).
   Prove: \(H\) is a subgroup of \(G\) under \(*\).
MAT 5420 S/S ’07 Assignment # 3

(Due at the beginning of class on Tuesday June 5, 2007)
Read §§1.4,2.2,3.2.
Hand in solutions to the following:
1. p. 112 problem 1
2. (a) Is \( \mathbb{Z}_{18}^* \) cyclic? Give full justification for your answer.
   (b) Is \( \mathbb{Z}_{24}^* \) cyclic? Give full justification for your answer.
3. Let \( G \) be a group. Let \( \sim \) denote the relation on \( G \) given by \( a \sim b \) if and only if there exists an element \( x \in G \) (possibly depending on \( a \) and \( b \)) such that \( b = xax^{-1} \).
   \textbf{Prove:} \( \sim \) is an equivalence relation on \( G \).

Also do, but do not hand in,
1. p. 112–114 problems 2,10,12,17
2. p. 43–44 problems 9,12
3. p. 70 problems 3,4,5
4. p. 70 problem 8 (replace \( \mathbb{Z} \) by \( \mathbb{Z} - \{0\} \))

MAT 5420 S/S ’07 Assignment # 4

(Due at the beginning of class on Tuesday June 12, 2007)
Read §2.3.
Hand in solutions to the following:
1. Let \( \sigma = \left( \begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 4 & 5 & 7 & 10 & 1 & 13 & 9 & 14 & 3 & 2 & 8 & 11 & 6 & 12 \end{array} \right) \in S_{14} \).
   (a) Express \( \sigma \) as a product of disjoint cycles.
   (b) Express \( \sigma \) as a product of transpositions.
   (c) Express \( \sigma^{-1} \) as a product of disjoint cycles.
   (d) Find the order of \( \sigma \).
2. p. 85 problem 14
3. p. 114 problem 19c,d (give proper justification for your answers)

Also do, but do not hand in,
p. 84,85 problems 1,3,4,8,10,11,12

Test Tuesday June 19
MAT 5420 S/S ’07 Assignment # 5

(Due at the beginning of class on Tuesday July 3, 2007)
Read §3.5,3.7
Hand in solutions to the following:
p.133 problem 10
p.141 problem 16 (do not assume that |G| is finite)
p.163 problem 7 (justify your answers)
Also do, but do not hand in,
p. 132–134 problems 2,4,5,9,13–16,18,24
p.141 problems 19,20
p.162–163 problems 4,6,8,9,10

MAT 5420 S/S ’07 Assignment # 6

(Due at the beginning of class on Tuesday July 10, 2007)
Read §3.8
Hand in solutions to the following:
1. p.176 problem 4
2. p.176 problem 10
3. p.176 problem 14 (Include an explanation of why N is a normal subgroup of G.)
Also do, but do not hand in,
p.176 problems 6,7,9,11,12,13,16

MAT 5420 S/S ’07 Assignment # 7

(Due at the beginning of class on Tuesday July 17, 2007)
Read §§5.1,5.2
Hand in solutions to the following:
1. p.234 problem 8
2. p.235 problem 14
3. p.249 problem 6 (You may use the fact that if a,b,c,d ∈ ℤ are such that a + b√2 = c + d√2, then a = c and b = d.)
Also do, but do not hand in,
p.234–235 problems 2,12,13a,16
p.249,250 problems 4,7,10,12,18

continued on next page
Read §5.3 and Chapter 4

Hand in solutions to the following:
1. p.259 problem 2
2. p.259 problem 10
3. p.260 problem 12 (The $R$ in parts (b) and (c) is the underlying ring of part (a).)

Also do, but do not hand in,
pp.259–261 problems 1,3,4,7,8,9,11,13,18,23,24

Test Tuesday July 31