MAT 1800 Final Exam

SHOW ALL WORK IN A BLUE BOOK: Only minimal credit will be awarded for answers without supporting work. DO NOT USE A CALCULATOR.

(08) 1. Let \( f(x) = \frac{x}{x - 1} - \frac{3}{x + 3} \) and \( g(x) = \frac{4}{x^2 + 2x - 3} \). Find all numbers \( x \) such that \( f(x) = g(x) \).

(08) 2. Graph: \( f(x) = \begin{cases} |x| - 1 & \text{if } x \leq 1 \\ \sqrt{x} & \text{if } 1 < x \leq 4 \\ 4 & \text{if } x > 4 \end{cases} \)

(08) 3. Find the domain of the function given by: \( \frac{\sqrt{x - 2}}{\ln(x - 3) - 1} + |x^2 - 1| \)

(08) 4. Let \( f(x) = x^2 + x \) and \( g(x) = \sqrt{x - 1} \). Find and simplify completely:

(a) \( \frac{(f \circ g)(2)}{f(2) \cdot g(2)} \) 
(b) \( \frac{f(x) - 2}{f(x - 2)} \)

(08) 5. Let \( f(x) = \sqrt{x + 3} \).

(a) Use the graph of \( f(x) \) to graph \( f^{-1}(x) \).

(b) Find \( f^{-1}(3) \).

(08) 6. For each part, give a function \( f \), by writing a formula for \( f(x) \), that satisfies the given conditions.

(a) The function \( f \) is a polynomial function of degree 4 with real coefficients for which \( x = i \) is a root.

(b) The function \( f \) is undefined at \( x = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2} \), ... and \( f(0) = 0 \).
7. Give a reasonable formula for each function whose graph is shown here:

(a) b)

8. The graph of a function $f$ is a line with $x$-intercept equal to $-2$ such that $2 \cdot f(0) = 8$. Find the function $f$.

9. A toy rocket is shot straight up in the air. The height in feet of the rocket $t$ seconds after being launched is given by the function $h(t) = -2t^2 + 60t$.

(a) How many seconds does it take for the rocket to reach its maximum height?

(b) What is the maximum height?

10. Graph $f(x) = (x + 3)(x - 1)^2$, finding and labeling all intercepts and asymptotes, if any.

11. Find all roots of the equation $3x^3 - 2x + 1 = 0$. Express any non-real roots in the form $a + bi$.

12. The resistance $R$ of a wire varies directly as its length $L$ and inversely as the square of its diameter $D$.

(a) Write an equation that expresses this joint variation.

(b) Find the constant of proportionality, if a wire $1200 \text{mm}$ long and $5 \text{mm}$ in diameter has a resistance of $144 \text{ ohms}$.

13. Graph $f(x) = \frac{x - 2}{x^2 - 4x}$, finding and labeling all intercepts and asymptotes, if any.
(08) 14. Let \( f(x) = e^{10 - 5x} - 4 \). Find all numbers \( x \), if there are any, such that \( f(x) = 16 \).

(08) 15. Graph \( f(x) = \ln(x + 1) - 2 \), finding and labeling all intercepts and asymptotes, if any.

(08) 16. Simplify completely:
   
   (a) \( e^{-2 \ln(\frac{1}{2})} \)
   
   (b) \( \log_8(6) - \log_8(3) - \log_8(2) \)

(08) 17. Solve: \( \log_3(-x) + \log_3(8 - x) - 2 = 0 \)

(08) 18. An investment grows exponentially according to the function \( P(t) = P_0e^{rt} \), where \( P(t) \) is the amount of money present after \( t \) years and \( P_0 \) is the initial amount. If it takes 5 years to double the investment, how long will it take for the investment to grow to 8 times the original amount?

(08) 19. Find the exact value, if it exists: (a) \( \csc(\frac{19\pi}{6}) \) (b) \( \tan(\frac{-9\pi}{4}) \)

(08) 20. Place each of the given numbers on the number line shown below. (Copy the number line on your answer sheet.)

   (a) \( \cos(6) \) (b) \( \ln(e + 0.01) \) (c) \( \sin(4.5) \) (d) \( \sin(6) \)

(08) 21. If \( \sin(\alpha) = \frac{2}{3}, \quad \frac{\pi}{2} < \alpha < \pi \) and \( \cos(\beta) = \frac{2}{3}, \quad -\frac{\pi}{2} < \beta < 0 \), find \( \sin(\alpha - \beta) \).

(08) 22. Let \( g(x) = -2 \sin(\frac{1}{2}x - \frac{\pi}{2}) \). Graph \( g \) over one complete cycle, labeling the highest and lowest points.

(08) 23. Find all primary solutions (i.e. \( 0 \leq x < 2\pi \)) of the equation \( \sqrt{3}\tan^2(x) - \tan(x) = 0 \)

(08) 24. Find the exact value:
   
   (a) \( \arcsin[\sin(-\frac{5\pi}{6})] \) (b) \( \cos[\arccos(\frac{\sqrt{3}}{2})] \)

(08) 25. Prove the identity: \( \frac{\sec(x) - \cos(x)}{\sec(x)} = \sin^2(x) \)