1800W021

MAT 1800 Final Exam

SHOW ALL WORK IN A BLUE BOOK: Only minimal credit will be awarded for answers without supporting work. DO NOT USE A CALCULATOR.

(08) 1. Let \( f(x) = x + \sqrt{x+1} \). Find all numbers \( x \) such that \( f(x) = 1 \).

(08) 2. Graph: \( f(x) = \begin{cases} 
1 - x & \text{if } x \leq -1 \\
|x| & \text{if } -1 < x < 0 \\
2 & \text{if } x \geq 0 
\end{cases} \)

(08) 3. Give a reasonable formula for each function whose graph is shown here:

(a)

(b)

(08) 4. Let \( f(x) = \frac{1}{5 + x} \) and \( g(x) = \frac{1}{x} \). Find and simplify completely:

(a) \( \frac{f(x + h) - f(x)}{h} \)  
(b) \( \frac{(g \circ f)(5)}{5} \)
5. Let $f$ be a function such that $f(2) = 3$ and $f(3) = 5$. Let $g$ be the inverse of $f$. Find and simplify: 
$(f \circ g)(4) + g(5)$

6. The graph of a function $f$ is a line perpendicular to the line passing through $(1, 1)$ and $(5, -1)$. And, $f(-2) = 7$. Find the function $f$.

7. A land owner wants to fence in a rectangular piece of waterfront property. She needs fencing on only 3 sides (as shown) and has a total of 18 yards of fencing.

(a) Write a function representing the area $A$ of the fenced in property as a function of the width $x$.

(b) Find the width that gives the maximum area.

8. Find the domain of the function given by: $f(x) = \frac{1}{1 + x} - \sqrt{9 - x^2}$

9. For each part, write a formula for a function $f$ which satisfies the given conditions:

(a) The function $f$ is a rational function with a horizontal asymptote of $y = 0$ and $f(0) = 3$.

(b) The function $f$ is a logarithmic function such that $f(9) = 2$.

10. Graph $f(x) = (x + 2)^2(x - 1)^2$, labeling all intercepts.

11. Given that $x = 2i$ is a root of the equation $x^4 - 2x^3 + 3x^2 - 8x - 4 = 0$, find all other roots. Express any non-real roots in the form $a + bi$.

12. Graph, finding and labeling all intercepts and asymptotes, if any: $f(x) = \frac{2x - 1}{x + 3}$

13. Let $f(x) = 2 \cdot 3^{x+1} - 1$. Find all numbers $x$, if there are any, such that $f(x) = 19$.

14. Graph, finding and labeling all intercepts and asymptotes, if any: $f(x) = e^{x+1} + 1$
15. Simplify completely:
(a) $\log_9(6) - \log_9(2)$
(b) $e^{\ln(5)} \cdot 3^{2 \log_3(7)}$

16. Solve: $\log_6(x - 3) = 1 - \log_6(x - 2)$

17. A certain substance is decaying exponentially according to the function $n(t) = n_0e^{-rt}$, where $n(t)$ is the amount of the substance present after $t$ years and $n_0$ is the initial amount. If the substance decays from 10,000 grams to 100 grams in two years, how many grams will there be after four years?

18. Find the exact value, if it exists: (a) $\cot \left( -\frac{17\pi}{6} \right)$ (b) $\sin \left( \frac{2\pi}{4} \right)$

19. Write an approximate value for each of the following numbers. Then arrange them from smallest on the left to largest on the right: $\ln(3)$, $\sin(2)$, $\sin(2.5)$, $\cos(2)$

20. If $\tan(x) = \sqrt{3}$ and $\csc(x) < 0$, find $\sin(2x)$.

21. Let $g(x) = \sin(2x - \frac{\pi}{2})$. Graph $g$ over one complete cycle, labeling the intercepts and the highest and lowest points.

22. Find all primary solutions (i.e. $0 \leq x < 2\pi$) of the equation: $\sin^2(x) = 2 - 3\cos^2(x)$

23. Find the exact value:
(a) $\cos^{-1}\left[ \cos\left( \frac{\pi}{3} \right) \right]$ (b) $\sin\left[ \sin^{-1}\left( \frac{1}{4} \right) \right]$

24. Prove the identity: $\frac{\cot^2(x)}{\csc(x)} = \csc(x) - \sin(x)$

25. Two straight roads, beginning at the same point $A$, form a 60° angle. A TransAm and a Taurus leave point $A$ at the same time, with the TransAm traveling on one road and the Taurus on the other. After one hour, the TransAm had traveled 80 miles and the Taurus 50 miles. At that time, how far apart were the cars?