MAT 1800 Final Exam

SHOW ALL WORK IN A BLUE BOOK: Only minimal credit will be awarded for answers without supporting work. DO NOT USE A CALCULATOR.

(08) 1. Let \( f(x) = 2x + \sqrt{5 - 4x} \). Find all numbers \( x \) such that \( f(x) = 1 \).

(08) 2. Graph: \( f(x) = \begin{cases} 
  x - 1 & \text{if } x \leq -2 \\
  |x| - 1 & \text{if } -2 < x < 3 \\
  2 & \text{if } x \geq 3
\end{cases} \)

(08) 3. Find the domain of the function given by: \( f(x) = \frac{\ln(x + 9)}{|x + 3| - 4} \)

(08) 4. Let \( f(x) = x^2 - 1 \) and \( g(x) = \sqrt{x + 3} \). Find and simplify completely:

\[
\begin{align*}
(\text{a}) & \quad \frac{(f \circ g)(1)}{f(1) + g(1)} \\
(\text{b}) & \quad \frac{f(x) - 3}{f(x + 3)}
\end{align*}
\]

(08) 5. The graph of a function \( f \) contains the following points: \((-1, 5)\), \((0, 7)\) and \((2, 12)\). The graphs of the functions \( g \), \( h \), \( j \) and \( k \) contain the points as listed. Which of the functions \( g \), \( h \), \( j \) and \( k \) could be \( f^{-1} \)?

\[
\begin{align*}
g : & \quad (5, -1), (3, -\frac{3}{2}), (0, -2) \\
h : & \quad (7, 0), (1, -2), (3, -2) \\
j : & \quad (7, 0), (-4, -4), (5, -2) \\
k : & \quad (0, -5), (3, -\frac{3}{2}), (1, -\pi)
\end{align*}
\]

(08) 6. For each part, give a function \( f \), by writing a formula for \( f(x) \), that satisfies the given conditions.

\[
\begin{align*}
(\text{a}) & \quad \text{The function } f \text{ is a root function with } f(8) = 2. \\
(\text{b}) & \quad \text{The function } f \text{ is undefined at } x = 0, \pm \pi, \pm 2\pi, \pm 3\pi \ldots \text{ and } f(\frac{x}{2}) = 1.
\end{align*}
\]
7. Give a reasonable formula for each function whose graph is shown here:

(a) 

(b) 

8. The graph of a function $f$ is a line such that $f(2) = 3$ and $f(2) + f(3) = 5$. Find the function $f$.

9. A rock is thrown straight up in the air. The height in meters of the rock $t$ seconds after being thrown is given by the function $h(t) = -5t^2 + 30t + 10$.
   (a) What is the maximum height reached by the rock?
   (b) How many seconds after being thrown does the rock reach this height?

10. Graph $f(x) = -x^3 + x^2 + 20x$, finding and labeling all intercepts and asymptotes, if any.

11. Given that $x^2 + 1$ is a factor of the polynomial $x^4 - 2x^3 - x^2 - 2x - 2$, find all roots of the equation $x^4 - 2x^3 - x^2 - 2x - 2 = 0$. Express any non-real roots in the form $a + bi$. 
12. The range $R$ of a projectile is directly proportional to the square of its velocity $V$. Suppose that the projectile has a range of 100 feet when it is traveling at a velocity of 30 feet/second.
   (a) Express $R$ as a function of $V$.
   (b) What is the velocity of a projectile whose range is 900 feet?

13. Graph $f(x) = \frac{x}{x^2 - 4}$, finding and labeling all intercepts and asymptotes, if any.

14. Let $f(x) = 4e^{-2x^3} - 6$. Find all numbers $x$, if there are any, such that $f(x) = 2$.

15. Graph $f(x) = e^{x^3} + 2$, finding and labeling all intercepts and asymptotes, if any.

16. Simplify completely:
   (a) $e^{-2\ln(\sqrt[3]{2})}$
   (b) $2\log_2(5) + 3\log_5(2^{10})$

17. Solve: $\ln(x - 4) - \ln(3) + \ln(x - 2) = 0$

18. A culture of bacteria grows exponentially according to the function $n(t) = n_0e^{t\ln(3)}$, where $n(t)$ is the quantity of bacteria present $t$ days after the initial observation and $n_0$ is the initial quantity. How long does it take the culture to grow to 9 times its original size? Simplify your answer.

19. Find the exact value, if it exists: (a) \(\cot(\frac{10\pi}{3})\) (b) \(\sin(-\frac{13\pi}{4})\)

20. Place each of the given numbers on the number line shown below. (Copy the number line on your answer sheet.)
   (a) $\ln(2.9)$  (b) $\sin(3.2)$  (c) $\cos(-6)$

21. If $\cot(\theta) = 5$ and $\sec(\theta) < 0$, find $\cos(2\theta)$.

22. Let $g(x) = -\tan(2x - \frac{\pi}{2})$. Graph $g$ over one complete cycle, labeling the intercepts and asymptotes, if any.

23. A point $P$ has polar coordinates $(2, \frac{4\pi}{3})$.
   (a) Find another polar representation of $P$.
   (b) Find the rectangular coordinates of $P$.

24. Find the exact value:
   (a) $\arcsin[\sin(\frac{5\pi}{6})]$  (b) $\sin[\arcsin(-\frac{\sqrt{3}}{2})]$\]

25. Prove the identity: $\csc(x) - \cot(x)\cos(x) = \sin(x)$